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A Formal Model of Neuron That Provides Consistent Predictions

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Abstract. We define maximal specific rules that avoid the problem of statistical ambiguity and provide predictions with maximum conditional probability. Also we define a special semantic probabilistic inference that learn these maximal specific rules and may be considered as a special case of Hebbian learning. This inference we present as a formal model of neuron and prove that this model provides consistent predictions.

Keywords: neuron, formal model, Hebbian learning, probabilistic inference.

1 Introduction

We had earlier suggested the formal model of neuron, which was based on semantic probabilistic inference [1-2]. This model was successfully tested by construction of animats [3].

In this work we show that this model allows us to solve the problem of statistical ambiguity and make consistent predictions.

The problem of statistic ambiguity consists in the following: during the process of learning (or inductive inference) we can get the probabilistic rules, which give us contradictions. This problem arises for the plenty of methods of machine learning. For instance, by observing the people, we can declare two following rules: if the man is a philosopher, then he is not a millionaire, and if he is a mine owner, then he is a millionaire. If there is a philosopher, who is also a mine owner, then we have a contradiction (as he is a philosopher, then he is not a millionaire, but as he is a mine owner, he is a millionaire). To get rid of these contradictions, Hempel [4] introduced the maximal specific requirement. Applying it to our example, we have that the following rules have to be the maximal specific ones: if the man is a philosopher but not a mine owner, then he is more likely not a millionaire, and if the man is a mine owner, but not a philosopher, then he is more likely a millionaire. It's not possible to use these two rules simultaneously, so there are no contradictions. Maximal specific rules must use all available information. In the next section we present the formal model of neuron that learns maximal specific rules.

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2 Description of the Formal Model of Neuron

Here we present informal description of the formal model of neuron, citing on the formal definitions given in the next section.

By *information,* given to brain as «input», we imply all stimulus provided by afferent system. Define the information, processed via nerve filament at neuron synapses, by single predicates $P_j^i(\mathbb{a}) = (x_i(\mathbb{a}) = x_{ij})$, $j = 1,...,n_i$, where $x_i(\mathbb{a})$ is an information, and x_{ij} is its value on the object (situation) $\mathbb{a}$. If this information transfer on excitatory synapse, then it perceived by neuron as a truth of the predicate $P_j^i(\mathbb{a})$, and if this information transfer to the inhibitory synapse, then it's been perceived as a negation of the predicate $\neg P_j^i(\mathbb{a})$.

We define the excitation of neuron (its axon) in a situation (object) $\mathbb{a}$ by a single predicate $P_0(\mathbb{a})$. If neuron is inhibited in a situation $\mathbb{a}$, then we define this as negation of the predicate $\neg P_0(\mathbb{a})$.

It is known, that each neuron does excite by its receptive field. This field is an initial (before training) semantics of the predicate $P_0(\mathbb{a})$. In the process of learning this information is enriched and can produce quite specific neurons as «Bill Clinton's neuron».

We suppose that formation of conditional reflex at the level of the neuron satisfy the Hebbian rule [5]. We developed a special semantic probabilistic inference [6-9] for formalization of the Hebbian rule in our model.

Predicates $P_j^i(\mathbb{a})$, $P_0(\mathbb{a})$ and their negations $\neg P_j^i(\mathbb{a})$, $\neg P_0(\mathbb{a})$ are literals, which we denote as $a, b, c, ... \in L$. In the process of semantic probabilistic inference neuron learn a set of rules {R} (conditional reflexes):

$$(a_1 \,\&\, ... \,\&\, a_k \Rightarrow b), \tag{1}$$

where $a_1,...,a_k$ are excitatory (inhibitory) predicates $P_j^i(\mathbb{a})$, $\neg P_j^i(\mathbb{a})$ and *b* is the predicate $P_0(\mathbb{a})$ or $\neg P_0(\mathbb{a})$.

Now we define a method for computing the conditional probability of the rule $(a_1 \,\&\, ... \,\&\, a_k \Rightarrow b)$. First we calculate the number of experiments $n(a_1,...,a_k,b)$ when the event $\langle a_1,...,a_k,b\rangle$ took place. Literally, this event means that immediately prior to the reinforcement there has been simultaneous excitation/inhibition of neuron inputs $\langle a_1,...,a_k\rangle$ and neuron itself. The reinforcement can be either positive or negative and be provided by motivation or emotion.

Among the cases $n(a_1,...,a_k,b)$ we calculate the cases $n^+(a_1,...,a_k,b)$ of positive reinforcements and $n^-(a_1,...,a_k,b)$ of the negative ones. The empirical conditional probability of the rule $(a_1 \,\&\, ... \,\&\, a_k \Rightarrow b)$ thus calculating as follows:

$$\mu(b / a_1,...,a_k) = n^+(a_1,...,a_k,b) - n^-(a_1,...,a_k,b) \big/ n(a_1,...,a_k,b).$$

If this probability negative, this means the inhibition of the neuron with probability taken with plus.

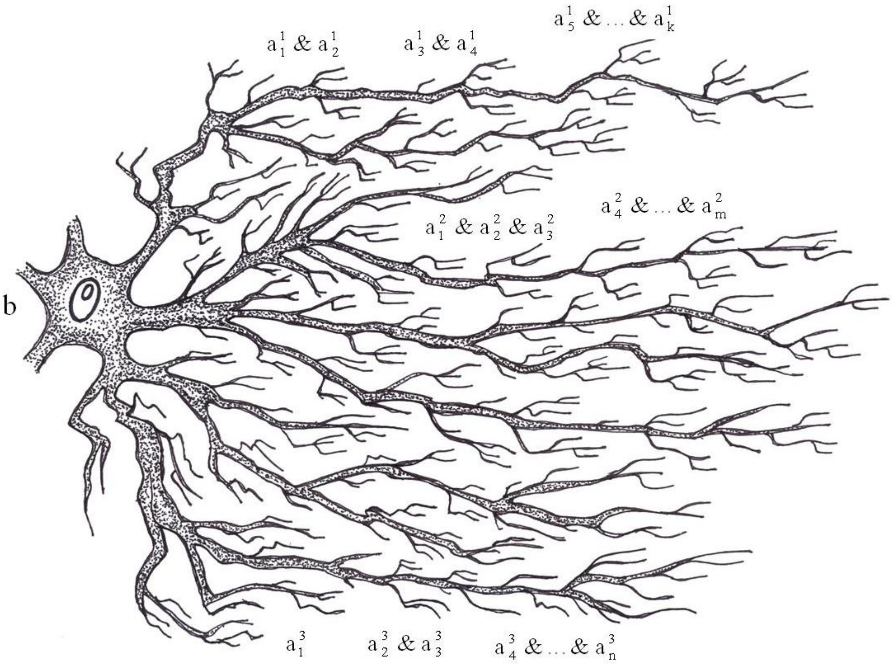

Fig. 1. Illustration of the semantic probabilistic inference on neuron

Formalization of the Hebbian rule by semantic probabilistic inference (definition 6) is performed in such a way that the following properties are satisfied:

1. if some conditional stimulus begin to predict neuron excitation by its receptive field with a certain probability, then conditional reflex at the level of neuron in the form of conditional rule (1) is learned by this neuron;
2. if new stimulus predict neuron excitation with even higher probability, then they are attached to this conditional rule. In this case we have the differentiation of conditional reflex. This differentiation is formalized in the notion of probabilistic inference (definition 5);
3. rules include only stimulus that are signal, i.e. each stimulus must increase the probability of the correct predictions for neuron excitation. This property is formalized as probabilistic law (definition 3);
4. excitation or inhibition of neuron via its set of rules {R} is executed by the rules with highest probability. This is confirmed by the fact that in the process of the conditional reflex learning, the speed of the neuron response on the conditional signal is higher for higher probability of this conditional reflex;
5. the rules with maximum probability are also maximal specific ones (definition 6), which use all available information. Thus, neuron turn to account all available information;

6. predictions, based on maximal specific rules are consistent in the limit (see theorem below). Thus, in the process of conditional reflex differentiation, neuron learn to predict without contradictions. It use either its excitatory maximal specific rules or the inhibitory ones (not simultaneously!);
7. for the formal model of neuron in fig. 1 there are some semantic probabilistic inferences. For instance, the rule $(b \Leftarrow a_1^1 \,\&\, a_2^1)$ is being strengthened by the new stimulus $a_3^1 \,\&\, a_4^1$ up to the rule $(b \Leftarrow a_1^1 \,\&\, a_2^1 \,\&\, a_3^1 \,\&\, a_4^1)$ if the stimulus $a_3^1 \,\&\, a_4^1$ increase the conditional probability of the excitation predictions for neuron b, and analogously the rule $(b \Leftarrow a_1^1 \,\&\, a_2^1 \,\&\, a_3^1 \,\&\, a_4^1)$ is being strengthened up to the rule $(b \Leftarrow a_1^1 \,\&\, a_2^1 \,\&\, a_3^1 \,\&\, a_4^1 \,\&\, a_5^1 \,\&\ldots\&\, a_k^1)$. The other two semantic inferences in the fig. 1 may be presented in the same way:

   a) $(b \Leftarrow a_1^2 \,\&\, a_2^2 \,\&\, a_3^2) \sqsubset (b \Leftarrow a_1^2 \,\&\, a_2^2 \,\&\, a_3^2 \,\&\, a_4^2 \,\&\ldots\&\, a_m^2)$;

   b) $(b \Leftarrow a_1^3) \sqsubset (b \Leftarrow a_1^3 \,\&\, a_2^3 \,\&\, a_3^3) \sqsubset (b \Leftarrow a_1^3 \,\&\, a_2^3 \,\&\, a_3^3 \,\&\, a_4^3 \,\&\ldots\&\, a_n^3)$.

The set of rules learned by neuron using semantic probabilistic inference, determines its formal model (definition 4), which predicts the excitation of neuron.

There are some other approaches to the probabilistic models of mind [10-11], but they are different from the semantic probabilistic inference [6-9].

3 Methods

Now we present the formal model description. By *data* we mean all situations of excitation or inhibition of a neuron in cases, when there was reinforcement. We denote the set of all rules of sort (1) by Pr.

Definition 1. The rule $R_1 = (a_1^1 \,\&\, a_2^1 \,\&\ldots\&\, a_{k_1}^1 \Rightarrow c)$ is *more general*, then the rule $R_2 = (b_1^2 \,\&\, b_2^2 \,\&\ldots\&\, b_{k_2}^2 \Rightarrow c)$ (we define this by $R_1 \succ R_2$) iff $\{a_1^1, a_2^1, \ldots, a_{k_1}^1\} \subset \{b_1^2, b_2^2, \ldots, b_{k_2}^2\}$, $k_1 < k_2$ and *no less general* $R_1 \gtrsim R_2$ iff $k_1 \le k_2$.

It's easy to show, that $R_1 \gtrsim R_2 \Rightarrow R_1 \vdash R_2$ and $R_1 \succ R_2 \Rightarrow R_1 \vdash R_2$, where $\vdash$ is a provability in propositional calculus.

We see that no less general (and more general) statements are logically stronger. Furthermore, more general rules are simpler because they contain smaller number of liters in the premise of the rule, so the relation $\succ$ can be perceived as the relation of simplicity in the sense of [12-13].

We define the set of sentences F, by the set of statements, obtained from the liters L by closure with respect to logic operations $\wedge, \vee$.

Definition 2. *Probability* on the set of sentences F is defined by the mapping $\mu : F \mapsto [0,1]$, such that [14]:

1. If $\vdash \varphi$, then $\mu(\varphi) = 1$;
2. If $\vdash \neg(\varphi \wedge \psi)$, then $\mu(\varphi \vee \psi) = \mu(\varphi) + \mu(\psi)$.

We define the conditional probability of the rule $R = (a_1 \& ... \& a_k \Rightarrow c)$ as

$$\mu(R) = \mu(c / a_1 \& ... \& a_k) = \frac{\mu(a_1 \& ... \& a_k \& c)}{\mu(a_1 \& ... \& a_k)}, \text{ if } \mu(a_1 \& ... \& a_k) > 0 .$$

We suppose that empirical conditional probability, calculated in the previous section, in the limit gives us μ. We define the set of all rules from $\Pr$, which conditional probability exists, by $\Pr_0$.

Definition 3. *Probabilistic law* is a rule $R \in \Pr_0$ that can't be logically strengthened without reducing its conditional probability, i.e. for every $R' \in \Pr_0$ if $R' \succ R$, then $\mu(R') < \mu(R)$.

Probabilistic laws are the most general, simple and logically strong rules. We define the set of all probabilistic laws by PL.

Definition 4. *Neuron formal model* is a set of all probabilistic laws $\Phi = \{R\}, R \in PL$, which are discovered by neuron.

Definition 5. *Probabilistic inference relation* $R_1 \sqsubseteq R_2$, $R_1, R_2 \in PL$ is defined by simultaneous fulfillment of two inequalities $R_1 \gtrsim R_2$ and $\mu(R_1) \le \mu(R_2)$. If both inequalities are strict, then the probabilistic inference relation is also *strict*

$$R_1 \sqsubset R_2 \Leftrightarrow R_1 > R_2 \,\&\, \mu(R_1) < \mu(R_2) .$$

Definition 6. *Semantic probabilistic inference* [6-9,13] is defined by the maximal (the one, we can't continue) sequence of probabilistic laws, which are in strict probabilistic inference relation $R_1 \sqsubset R_2 \sqsubset ... \sqsubset R_k$. The last probabilistic law R_k of this inference is a *maximal specific* one.

Theorem [7]. Predictions, based on maximal specific rules, are consistent: it is impossible to obtain a contradiction (ambiguity) using *maximal specific* rules, i.e. there are no exist two maximal specific rules such that $(a_1 \& ... \& a_k \Rightarrow c)$, and $(b_1 \& ... \& b_l \Rightarrow \neg c), \mu(b_1 \& ... \& b_l \& a_1 \& ... \& a_k) > 0$.

We have developed the programming system Discovery, which realizes semantic probabilistic inference and had been successfully applied for solution of several applied tasks [15-16].

4 Conclusion

The formal model of neuron, on the one hand, formalizes the Hebbian rule and, on the other hand, allows us to make a consistent predictions.

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