

Conceptually, law-like rules came from the philosophy of science. These rules attempt to mathematically capture the essential features of scientific laws:

- (1) high level of generalization;
- (2) simplicity (Occam's razor); and,
- (3) refutability.

Formally, an IF-THEN rule  $C$  is presented as  $A_1 \& \dots \& A_k \Rightarrow A_0$ , where the IF-part,  $A_1 \& \dots \& A_k$ , consists of true/false logical statements  $A_1, \dots, A_k$ , and the THEN-part consists of a single logical statement  $A_0$ . Rule  $C$  allows one to generate sub-rules with a truncated IF-part, e.g.  $A_1 \& A_2 \Rightarrow A_0$ ,  $A_1 \& A_2 \& A_3 \Rightarrow A_0$  and so on. It is known that a sub-rule is logically stronger than the rule used to construct the sub-rule. Thus, if some rule and its sub-rule  $C$  classify correctly the same set of examples, then the sub-rule is preferred.

In general, there are three reasons to prefer the sub-rule:

1. The sub-rule is more general (logically stronger and describes the same set of events).
2. The sub-rule is simpler than the rule, because it consists of fewer statements in the IF-part.
3. Sub-rule is better testable (more refutable) than the rule, because the larger set of possible examples may falsify it (the IF-part of the sub-rule is less restrictive).

Thus, if a rule covers the set of examples then one can test that no one of its sub-rules also covers the same set of examples. Otherwise, this sub-rule or maybe some of its sub-rules will be preferred, because this sub-rule is simpler, more general and more refutable.

In deterministic case, a **law-like** rule can be defined (for some set of examples) as a rule without sub-rules covering this set of examples. In other words **law-like** rule is the rule which is true for some set of examples, but no one of its sub-rule is true for this data.

If examples contain noise, which is typical in many fields, the probabilistic characteristics of the expressions are used instead of crisp (true/false) values. The conditional probability of the rule is used in the MMDR method as this characteristic. For rule  $C$ , its conditional probability  $\text{Prob}(C) = \text{Prob}(A_0/A_1 \& \dots \& A_k)$  is defined, assuming that  $\text{Prob}(A_1 \& \dots \& A_k) > 0$ . Similarly conditional probabilities  $\text{Prob}(A_0/A_i1 \& \dots \& A_ih)$  are defined for sub-rules  $C_i$ , such as  $A_i1 \& \dots \& A_ih \Rightarrow A_0$ , assuming that  $\text{Prob}(A_i1 \& \dots \& A_ih) > 0$ . Conditional probability,  $\text{Prob}(C) = \text{Prob}(A_0/A_1 \& \dots \& A_k)$ , is used for estimating forecasting power of the rule to predict  $A_0$ . In addition, the conditional probability is a major tool for defining non-deterministic (probabilistic) law-like rules (regularities) (Vit-yaev E et al. 1998;1992).

The rule is a probabilistic **law-like** rule iff all of its sub-rules have a statistically significant lower conditional probability than the rule. Another definition of **law-like** rules can be given in terms of generalization. The rule is **law-like** iff it can not be generalized without producing a statistically significant reduction in its conditional probability. Law-like rules defined in this way hold all three listed above properties (properties of scientific laws), i.e., these rules are:

- (1) general from a logical perspective,
- (2) simple, and
- (3) refutable.