

The main definitions from representative measurement theory are reviewed in this section. A relational structure A consists of a set A and relations S_1, \dots, S_n defined on A

$$A = \langle A, S_1, \dots, S_n \rangle.$$

Each relation S_i is a Boolean function (predicate) with n_i arguments from A . The relational structure $A = \langle A, S_1, \dots, S_n \rangle$ is considered along with a relational structure of the same type

$$R = \langle R, T_1, \dots, T_n \rangle.$$

Usually the set R is a subset of Re^m , $m \geq 1$, where Re^m is a set of m -tuples of real numbers and each relation T_i has the same n_i as the corresponding relation S_i . T_i and S_i are called a k -ary relation on R . Theoretically, it is not a formal requirement that R be numerical.

Next, the relational system A is interpreted as an empirical real-world system and R is interpreted as a numerical system designed as a numerical representation of A . To formalize the idea of numeric representation, we define a homomorphism φ as a mapping from A to R .

A mapping $\varphi: A \rightarrow R$ is called a homomorphism if for all i ($i = 1, \dots, n$),

$$(a_1, \dots, a_{k(i)}) \in S_i \Leftrightarrow (\varphi(a_1), \dots, \varphi(a_{k(i)})) \in T_i.$$

In other notation,

$$S_i(a_1, \dots, a_{k(i)}) \Leftrightarrow T_i(\varphi(a_1), \dots, \varphi(a_{k(i)})).$$

Let $\Phi(A, R)$ be the set of all homomorphisms for A and R . It is possible that $\Phi(A, R)$ is empty or contains a variety of representations. Several theorems are proved in RMT about the contents of $\Phi(A, R)$. These theorems involve: (1) whether $\Phi(A, R)$ is empty, and (2) the size of $\Phi(A, R)$. The first theorems are called representation theorems. The second theorems are called uniqueness theorems.

Using the set of homomorphisms $\Phi(A, R)$ we can define the notion of permissible transformations and the data type (scale types). The most natural concept of permissible transformations is a mapping of the numerical set R into itself, which should bring a “good” representation. More precisely, γ is permissible for $\Phi(A, R)$ if γ maps R into itself, and for every φ in $\Phi(A, R)$, $\gamma\varphi$ is also in $\Phi(A, R)$. For instance, the permissible transformations could be transformations, $x \rightarrow rx$ or monotone transformations $x \rightarrow \gamma(x)$.