1 Mathematical model

Assume that the set I of potential facility locations and the set J of clients are finite. For each facility $i \in I$ we have the set R_i of design scenarios and this set is finite as well. For each pair $i \in I, r \in R_i$ we have the fixed costs f_{ir} and g_{ir} of opening facility i with design scenario r by the leader and by the follower, respectively. Moreover, we know the attractiveness a_{ir} of the leader facility and the similar parameter b_{ir} of the follower facility. The last two features are important for describing the client behavior. Each client j splits own demand w_j probabilistically over all facilities directly proportional with attraction to each facility and inversely proportional to the distance d_{ij} between client j and facility i. Following [1], we consider the utility function u_{ijr} of leader facility i with design scenario r for client j and the similar function v_{ijr} for follower facility:

$$u_{ijr} = a_{ir}/(d_{ij}+1)^{\beta}, \quad v_{ijr} = b_{ir}/(d_{ij}+1)^{\beta}, \qquad i \in I, r \in R_i, j \in J,$$

where β is a distance sensitivity parameter. Now we introduce the decision variables for the players:
x_{ir} is equal to 1 if facility i is open by the leader with design scenario r and 0 otherwise;
y_{ir} is equal to 1 if facility i is open by the follower with design scenario r and 0 otherwise.
For client j, the total utility U_j from the leader facilities and the total utility V_j from the follower facilities are defined as:

$$U_j = \sum_{i\in I}\sum_{r\in R_i} u_{ijr}x_{ir}, \quad V_j = \sum_{i\in I}\sum_{r\in R_i} v_{ijr}y_{ir}, \qquad j \in J.$$

The total market share of the leader is given by $\sum_{j\in J} w_j U_j/(U_j+V_j)$. The leader wishes to maximize own market share, anticipating that the follower will react to the decision by opening own facilities. The market share of the follower is given by $\sum_{j\in J} w_j V_j/(U_j+V_j)$. The follower maximizes own market share. In opposite [2], we assume that the players can open facilities at the same site. This Stackelberg game can be presented as the following nonlinear 0–1 bilevel optimization problem [3]:

$$\max_x \sum_{j\in J} w_j \frac{\sum_{i\in I}\sum_{r\in R_i} u_{ijr}x_{ir}}{\sum_{i\in I}\sum_{r\in R_i} u_{ijr}x_{ir} + \sum_{i\in I}\sum_{r\in R_i} v_{ijr}y^*_{ir}} \qquad (1)$$

subject to

$$\sum_{i \in I} \sum_{r \in R_i} f_{ir} x_{ir} \leq B_l; \tag{2}$$

$$\sum_{r \in R_i} x_{ir} \leq 1, \qquad i \in I; \tag{3}$$

$$x_{ir} \in \{0, 1\}, \qquad r \in R_i, i \in I; \tag{4}$$

where y^*_{ir} is the optimal solution for the follower problem:

$$\max_y \sum_{j \in J} w_j \frac{\sum_{i \in I} \sum_{r \in R_i} v_{ijr} y_{ir}}{\sum_{i \in I} \sum_{r \in R_i} u_{ijr} x_{ir} + \sum_{i \in I} \sum_{r \in R_i} v_{ijr} y_{ir}} \tag{5}$$

subject to

$$\sum_{i \in I} \sum_{r \in R_i} g_{ir} y_{ir} \leq B_f; \tag{6}$$

$$\sum_{r \in R_i} y_{ir} \leq 1, \qquad i \in I; \tag{7}$$

$$y_{ir} \in \{0, 1\}, \qquad r \in R_i, i \in I. \tag{8}$$

Objective functions (1) and (5) are market shares of the players. Inequalities (2) and (6) are the budget constraints: B_l is the budget of the leader, B_f is the budget of the follower. Inequalities (3) and (7) ensure the only design scenario for each open facility.

References

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[3] Yu. Kochetov, N. Kochetova, and A. Plyasunov. A matheuristic for the leader-follower facility location and design problem. *10th international Metaheuristics Conference (MIC 2013).* Singapure 5-8 August, 2013.