International Conference on Ring Theory
Shirshov-2021
dedicated to the 100th anniversary
of A. I. Shirshov (1921-1981)

Organizers:

Sobolev Institute of Mathematics,

Mathematical Center in Akademgorodok

Units of Integral Group Rings of Cyclic 2-groups

Rifkhat Zh. Aleev [1]
South Ural State University (National Research University),
Chelyabinsk State University
aleevrz@susu.ru

We introduce the following notation. Let $n \geqslant 3$. Then

1) α be a primitive complex 2^nth root of unity,

2) for any integer j, we put $d_j = 1 + \alpha^j + \alpha^{-j}$.

1 Unit group of the ring $\mathbf{Z}[\alpha]$

Lemma 1 *Let* $\mathrm{Un}\,(\mathbf{Z}[\alpha])$ *be a unit group of* $\mathbf{Z}[\alpha]$*. Then*

$$\mathrm{Un}\,(\mathbf{Z}[\alpha]) = \langle\alpha\rangle \times K,$$

where $K \subset \mathrm{Un}(\mathbf{Z}[\alpha + \alpha^{-1}]) \subset \mathbf{R}$*.*

Let $P = \left\langle 1 - \alpha^k \mid k \in \{1, 2 \ldots, 2^n - 1\} \right\rangle \leqslant \mathbf{Q}^*_{2^n}$ be a subgroup of multiplicative group $\mathbf{Q}^*_{2^n}$ of the cyclotomic field $\mathbf{Q}_{2^n}$. Then we call

$$K(\alpha) = P \cap \mathrm{Un}\,(\mathbf{Z}[\alpha])$$

by the *circular unit group* of the field $\mathbf{Q}_{2^n}$.

Lemma 2 ([2]) *Let* $h(n)$ *be a class number of the field* $\mathbf{Q}_{2^n} \cap \mathbf{R}$*. Then*

$$|\mathrm{Un}\,(\mathbf{Z}[\alpha]) : K(\alpha)| = h(n).$$

Put

$$D = \prod_{l=0}^{2^{n-2}-2} \langle d_{2l+1} \rangle = \langle d_1 \rangle \times \langle d_3 \rangle \times \cdots \times \langle d_{2^{n-1}-3} \rangle\,.$$

Theorem 1 *We have*

$$K(\alpha) = \langle\alpha\rangle \times D.$$

As a corollary of the theorem 1, we obtain.

Corollary 1 $K(\alpha) \cap \mathbf{R} = \langle -1 \rangle \times D$*.*

[1]The reported study was funded by RFBR and BRFBR, project number 20-51-00007

2 Direct decomposition

Let $G = \langle x \rangle$ be a cyclic group of order 2^b. Let χ_i be the character of the group G with $\chi_{2^i}(x) = \alpha^{2^i}$ for every $i \in \{0, 1, \dots, n-1\}$. The local units $u_{\chi_{2^i}}(\beta)$ for $\beta \in \mathrm{Un}(\mathbf{Z}[\alpha^{2^i}]$ are defined according to [1, Definition 1].

Let's define a subgroup W_1 of the normalized unit group $\mathrm{V}(\mathbf{Z}G)$ of an integral group ring $\mathbf{Z}G$ for a cyclic group G as follows:

$$W_1 = \langle u_{\chi_1}(\beta_1) \mid \beta_1 \in \mathrm{Un}(\mathbf{Z}[\alpha]) \rangle \,,$$

The local units are a multiplicative, and so

$$W_1 = \{ u_{\chi_1}(\beta_1) \mid \beta_1 \in \mathrm{Un}(\mathbf{Z}[\alpha]) \} \,.$$

Theorem 2 *Let $\beta \in \mathrm{Un}(\mathbf{Z}[\alpha])$. Local unit $u_{\chi_1}(\beta) \in \mathrm{V}(\mathbf{Z}G)$ if and only if*

1) *$\beta \in \mathrm{Un}(\mathbf{Z}[\alpha + \alpha^{-1}]) = \langle -1 \rangle \times K$, where K as in Lemma 1,*

2) *with $\beta \equiv 1 \pmod 2$.*

Next, we will limit ourselves to considering *only circular units*. More precisely, only the elements of the group Dwill be considered.

We introduce the following notation.

1. $E = \{ \lambda \in D \mid \lambda \equiv 1 \pmod 2 \} = (1 + 2\mathbf{Z}[\alpha]) \cap D$.
2. $V_1 = \{ u_{\chi_1}(\lambda) \mid \lambda \in E \}$.

We obtain a description of the structure of the group W_1.

Corollary 2

1. *V_1 is the subgroup of the group W_1.*
2. *$W_1 = \langle x^{2^{n-1}} \rangle \times V_1$.*

Lemma 3

1. $\left| D : D^{2^{n-2}} \right| = 2^{(n-2)(2^{n-2}-1)}$.
2. *E is the subgroup of D and $D^{2^{n-2}} \leqslant E$.*

Let

$$V_2 = \left\{ \prod_{i=1}^{n-1} u_{\chi_{2^i}}(\beta_i) \mid \beta_i \in \mathrm{Un}(\mathbf{Z}[\alpha^{2^i}], i \in \{1, \dots, n-1\} \right\}.$$

Theorem 3 *We have:*

1) *V_2 is a torsion free subgroup of $\mathrm{V}(\mathbf{Z}G)$,*

2) *V_2 is isomorphic to subgroup of $\mathrm{V}(\mathbf{Z}\langle x^2 \rangle)$,*

3) *$\langle x \rangle \times V_1 \times V_2$ is a subgroup of $\mathrm{V}(\mathbf{Z}G)$ of finite index.*

So we can realize inductive transfer from $\mathrm{V}(\mathbf{Z}\langle x^2 \rangle)$ to $\mathrm{V}(\mathbf{Z}G)$ up to subgroup of finite index.

3 Group F

From theorem 3 we have it is important to study subgroup V_1, or equivalent E.

Put

$$A_0 = \{1, 3, 5, \dots, 2^{n-2} - 1\} = \left\{2l + 1 \mid l \in \{0, \dots, 2^{n-3} - 1\}\right\},$$
$$B_0 = A \setminus A_0 = \left\{2^{n-1} - (2l + 1) \mid 2l + 1 \in A_0 \setminus \{1\}\right\}.$$

For any $k \in \{1, \dots, n-3\}$ we put

$$A_k = \{1, 3, 5, \dots, 2^{n-2-k} - 1\} = \left\{2l + 1 \mid l \in \{0, \dots, 2^{n-3-k} - 1\}\right\},$$
$$B_k = A_{k-1} \setminus A_k = \left\{2^{n-1-k} - (2l + 1) \mid 2l + 1 \in A_k\right\}.$$

For any $2l + 1 \in \left\{3, \dots, 2^{n-2} - 1\right\} = A_0 \setminus \{1\}$ we put

$$q(0, 2l + 1) = d_{2l+1}^{-1} d_{2^{n-1}-(2l+1)}.$$

Let $k \in \{1, \dots, n-3\}$. For any $2l + 1 \in \left\{1, \dots, 2^{n-2-k} - 1\right\} = A_k$ we put

$$q(k, 2l + 1) = d_{2l+1}^{-1} d_{2^{n-1-k}-(2l+1)}.$$

Lemma 4

$$D = \langle d_1 \rangle \times \prod_{2l+1 \in A_0 \setminus \{1\}} \langle q(0, 2l + 1) \rangle \times \prod_{k=1}^{n-3} \prod_{2l+1 \in A_k} \langle q(k, 2l + 1) \rangle.$$

For the step 0, we put

$$F_0 = \prod_{2l+1 \in A_0 \setminus \{1\}} \langle d_{2l+1}^{-1} d_{2^{n-1}-(2l+1)} \rangle = \prod_{2l+1 \in A_0 \setminus \{1\}} \langle q(0, 2l + 1) \rangle.$$

At step $k \in \{1, 2, \dots, n-3\}$ we put

$$F_k = \prod_{2l+1 \in A_k} \langle d_{2l+1}^{-2^k} d_{2^{n-2}-(2l+1)}^{2^k} \rangle = \prod_{2l+1 \in A_k} \langle q(k, 2l + 1)^{2^k} \rangle.$$

Finally,

$$F = \langle d_1^{2^{n-2}} \rangle \times \prod_{k=0}^{n-3} F_k.$$

Theorem 4 *F is a subgroup in E. In addition,*

$$D^{2^{n-2}} < F \leqslant E < D,$$

and, moreover, for $n \in \{2, 4, 4, 6, 7\}$

$$F = E.$$

References

[1] R. Zh. Aleev, Units of character fields and central units of integral group rings of finite fields, *Sib. Adv. of Math.* **11** (2001), 1–33.

[2] W. Sinnott, On the Stickelberger ideal and circular units of a cyclotomic field. *Ann. of Math.* **108** (1978) 107–134.

On Property (A) of rings and modules over an ideal

Arssi Youssef (University of Moulay Ismail, Meknes, Morocco)

In this work, we introduces and studies the notion of Property (A) of a ring R or an R-module M along an ideal I of R. For instance, any module M over R satisfying the Property (A) do satisfy the Property (A) along any ideal I of R. We are also interested in ideals I which are A-module along themselves. In particular, we prove that if I is contained in the nilradical of R, then any R module is an A-module along I and, thus, I is an A-module along itself. Also, we present an example of a ring R possessing an ideal I which is an A-module along itself while I is not an A-module. In particular, we prove that if I is contained in the nilradical of R, then any R module is an A-module along I and, thus, I is an A-module along itself. Also, we present an example of a ring R possessing an ideal I which is an A-module along itself while I is not an A-module.

Transposed Poisson algebras

Chengming Bai (Nankai University, China)

We introduce a notion of transposed Poisson algebra which is a dual notion of the Poisson algebra by exchanging the roles of the two binary operations in the Leibniz rule defining the Poisson algebra. We interpret the close relationships between it and some structures such as Novikov-Poisson and pre-Lie Poisson algebras including the example given by a commutative associative algebra with a derivation, and 3-Lie algebras.

MINIMAL VARIETIES OF ASSOCIATIVE ALGEBRAS AND TRANSCENDENTAL SERIES

VESSELIN DRENSKY

A variety of associative algebras over a field of characteristic 0 is called minimal if the exponent of the variety which measures the growth of its codimension sequence is strictly larger than the exponent of any of its proper subvarieties, i.e. its codimension sequence grows much faster than the codimension sequence of its proper subvarieties. By the results of Giambruno and Zaicev it follows that the number b_n of minimal varieties of given exponent n is finite. Using methods of the theory of colored (or weighted) compositions of integers, we show that the limit $\beta = \lim_{n\to\infty} \sqrt[n]{b_n}$ exists and can be expressed as the positive solution of an equation $a(t) = 0$ where $a(t)$ is an explicitly given power series. Similar results are obtained for the number of minimal varieties with a given Gelfand-Kirillov dimension of their relatively free algebras of rank d. It follows from classical results on lacunary power series that the generating function of the sequence b_n, $n = 1, 2, \ldots$, is transcendental. With the same approach we construct examples of free graded semigroups $\langle Y\rangle$ with the following property. If d_n is the number of elements of degree n of $\langle Y\rangle$, then the limit $\delta = \lim_{n\to\infty} \sqrt[n]{d_n}$ exists and is transcendental.

The talk is based on the paper
V. Drensky, Minimal varieties of associative algebras and transcendental series, International J. Algebra and Computation 31 (2021), No. 2, 241-256.

Institute of Mathematics and Informatics, Bulgarian Academy of Sciences, 1113 Sofia, Bulgaria

E-mail address: drensky@math.bas.bg

GRADED-DIVISION ALGEBRAS AND GALOIS EXTENSIONS

ALBERTO ELDUQUE

Abstract. Graded-division algebras are building blocks in the theory of finite-dimensional associative algebras graded by a group G. If G is abelian, they can be described, using a loop construction, in terms of central simple graded-division algebras.

On the other hand, given a finite abelian group G, any central simple G-graded-division algebra over a field $\mathbb{F}$ is determined, thanks to a result of Picco and Platzeck, by its class in the (ordinary) Brauer group of $\mathbb{F}$ and the isomorphism class of a G-Galois extension of $\mathbb{F}$.

This connection is used to classify the simple G-Galois extensions of $\mathbb{F}$ in terms of a Galois field extension with Galois group isomorphic to a quotient G/K plus some extra structure. Non-simple G-Galois extensions are induced from simple T-Galois extensions for a subgroup T of G.

This is a joint work with Mikhail Kochetov.

References

[EK21] A. Elduque and M. Kochetov, *Graded division algebras and Galois extensions*, J. Pure Appl. Algebra **221** (2021), 106773.

Departamento de Matemáticas e Instituto Universitario de Matemáticas y Aplicaciones, Universidad de Zaragoza, 50009 Zaragoza, Spain

Email address: elduque@unizar.es

On finiteness properties of some subgroups of Aut(F_n)

Ershov Mikhail (University of Virginia)

Let F_n denote a free group on n generators. In 1920s Nielsen proved that its automorphism group Aut(F_n) is finitely generated. In 1935 Magnus established finite generation for IA_n, the Torelli subgroup of Aut(F_n), which is defined as the kernel of the natural map from Aut(F_n) to GL_n(Z). After briefly discussing these theorems, I will talk about recent results on finite generation of some terms of the lower central series of IA_n. If time allows, I will also discuss the corresponding results for the Torelli subgroups of the mapping class groups. The talk will be based on joint works with Sue He, with Thomas Church and Andrew Putman, and with Daniel Franz.

Lie algebras of vector fields on algebraic varieties

Vyacheslav Futorny

(University of São Paulo, Brasil)

Abstract: Lie algebras of polynomial vector fields on algebraic varieties are defined as derivation algebras of the rings of functions on affine algebraic varieties. Classical examples include the Virasoro and the Witt algebras which correspond to the cases of circle and torus respectively and whose representation theory is well understood. We will discuss state of the art of the representation theory in the case of arbitrary varieties based on recent results with Y.Billig, J.Nilsen and A.Zaidan.

Double Lie algebras of a nonzero weight

Maxim Goncharov (a joint work with Vsevolod Gubarev)
Sobolev Institute of Mathematics, Novosibirsk State University
arXiv:2104.13678

The notion of a **double Poisson algebra** on a given associative algebra was introduced by M. Van den Bergh in 2008 [3] as a noncommutative analog of Poisson algebra. Double Poisson algebra is an associative algebra A equipped with a linear bracket $\{\{\cdot,\cdot\}\} : A \otimes A \mapsto A \otimes A$, satisfying the analogs of anti-commutativity, Jacobi identity, and Leibniz rule.

In the middle of 2010s, S. Arthamonov introduced a notion of **modified double Poisson algebra** [1, 2] with weakened versions of anti-commutativity and Jacobi identity. This notion allowed S. Arthamonov to study the Kontsevich system and give more examples of H_0-Poisson structures arisen from double brackets.

The notion of **double Lie algebra** naturally arose directly from the definition of double Poisson algebra, it is a vector space V endowed with a double bracket satisfying anti-commutativity and Jacobi identity mentioned above, and we forget about associative product on V.

It is known that double Lie algebras on a finite-dimensional vector space V are in one-to-one correspondence with skew-symmetric Rota—Baxter operators of weight 0 on the matrix algebra $M_n(F)$, where $n = \dim(V)$ [4]. A linear operator R defined on an algebra A is called a **Rota—Baxter operator** (RB-operator, for short) of weight λ, if

$$R(x)R(y) = R(R(x)y + xR(y) + \lambda xy)$$

for all $x, y \in A$.

We apply Rota—Baxter operators of nonzero weight on the matrix algebra to define a weighted analog of double Lie algebras. We define what is a λ-double Lie algebra for a fixed $\lambda \in F$. Thus, in the finite-dimensional case we extend the bijections

$$\text{double Lie a.} \iff \begin{array}{c}\text{skew-symmetric}\\ \text{RB-operator of w. } 0 \text{ on } M_n(F)\end{array} \iff \begin{array}{c}\text{skew-symmetric}\\ \text{sol. of AYBE on } M_n(F)\end{array}$$

for the weighted analogs of the objects as follows,

$$\lambda\text{-double Lie a.} \iff \begin{array}{c}\lambda\text{-skew-symmetric}\\ \text{RB-operator of w. } \lambda \text{ on } M_n(F)\end{array} \iff \begin{array}{c}(-\lambda)\text{-skew-symmetric}\\ \text{sol. of AYBE}(-\lambda) \text{ on } M_n(F)\end{array}$$

The correspondence between λ-double Lie algebras and RB-operators of weight λ is helpful for constructing examples of λ-double Lie algebras. As in the case $\lambda = 0$, we prove that there are no simple finite-dimensional λ-double Lie algebras.

Finally, we prove that every λ-double Lie algebra structure on a vector space V generates a unique modified double Poisson algebra structure on $\mathrm{As}\langle V\rangle$. This general result confirms the conjecture of S. Arthamonov (2017) [2], which says that the double bracket defined on the three-dimensional vector space $V = \mathrm{Span}\{a_1, a_2, a_3\}$ as follows,

$$\{\!\{a_1, a_2\}\!\} = -a_1 \otimes a_2, \quad \{\!\{a_2, a_1\}\!\} = a_1 \otimes a_2, \quad \{\!\{a_2, a_3\}\!\} = a_3 \otimes a_2,$$
$$\{\!\{a_3, a_1\}\!\} = a_1 \otimes a_3 - a_3 \otimes a_1, \quad \{\!\{a_3, a_2\}\!\} = -a_3 \otimes a_2$$

can be extended to a modified double Poisson algebra structure on $\mathrm{As}\langle a_1, a_2, a_3\rangle$.

References

[1] S. Arthamonov, Noncommutative inverse scattering method for the Kontsevich system, Lett. Math. Phys. (9) **105** (2015) 1223–1251.

[2] S. Arthamonov, Modified double Poisson brackets, J. Algebra **492** (2017) 212–233.

[3] M. Van den Bergh, Double Poisson algebras, Trans. Amer. Math. Soc. **360** (11) (2008) 5711–5769.

[4] M. E. Goncharov, P. S. Kolesnikov. Simple finite-dimensional double algebras, J. Algebra **500** (2018) 425–438.

Rota–Baxter operators on different algebraic structures

Gubarev Vsevolod (Sobolev Institute of Mathematics, Russia)

The notion of the Rota–Baxter operator defined on an algebra is known since 1951 [4]. In the last few years, the analogues of this notion for different algebraic structures have appeared:

- Rota–Baxter operator on conformal algebra (Y. Hong, C. Bai. 2020 [3]);
- Rota–Baxter operator on group (L. Guo, H. Lang, Y. Sheng, 2021 [2]);
- Rota–Baxter operator on Hopf algebra (M. Goncharov, 2021 [1]).

We discuss these new objects, provide their examples and study their connections.

Bibliography

[1] M. Goncharov, Rota–Baxter operators on cocommutative Hopf algebras, J. Algebra **582** (2021), 39–56.
[2] L. Guo, H. Lang, Y. Sheng, Integration and geometrization of Rota–Baxter Lie algebras, Adv. Math. **387** (2021), #107834, 34 pp.
[3] Y. Hong, C. Bai, Conformal classical Yang–Baxter equation, S-equation and O-operators, Lett. Math. Phys. **110** (2020), 885–909.
[4] F.G. Tricomi, On the finite Hilbert transform, Quart. J. Math. **2** (1951) 199–211.

Noncommutative Poisson structures, Hochschild type complexes and Groebner-Shirshov bases theory

Natalia Iyudu (University of Edinburgh, UK)

I will explain how pre-Calabi-Yau structures give rise to double Poisson brackets of Van den Bergh. The horological formulation of pre-Calabi-Yau structure can be dealt with using Groebner-Shirshov bases theory to prove purity and formality in case of graph path algebras.

Evaluations of nonassociative polynomials on finite dimensional algebras

Alexei Kanel-Belov (Bar-Ilan University (Israel))
and Sergey Malev (Ariel University of Samaria (Israel))

Let p be a polynomial in several non-commuting variables with coefficients in an algebraically closed field K of arbitrary characteristic. It has been conjectured that for any n, for p multilinear, the image of p evaluated on the set $M_n(K)$ of n by n matrices is either zero, or the set of scalar matrices, or the set $sl_n(K)$ of matrices of trace 0, or all of $M_n(K)$.

In this talk we will discuss the generalization of this conjecture for non-associative algebras such as Cayley-Dickson algebra (i.e. algebra of octonions), pure (scalar free) octonion Malcev algebra and basic low rank Jordan algebras.

Alexei Kanel-Belov, Sergey Malev, Louis Rowen, Roman Yavich, “Evaluations of noncommutative polynomials on algebras: Methods and problems, and the Lvov-Kaplansky Conjecture”, Symmetry Integrability Geom. Methods Applications,, 16 (2020), 071 , 61 pp., Special Issue on Algebra, Topology, and Dynamics in Interaction in honor of Dmitry Fuchs.

Drinfeld-Jimbo quantizations as Koszul algebras

V.K. Kharchenko (National Autonomous University of Mexico, Mexico)

The Koszul algebras arise in many areas of the modern mathematics: algebraic geometry, representation theory, noncommutative geometry, topology, number theory, theory of pseudoroots of noncommutative polynomials. We prove that in q-Weyl generators the multi-parameter Drinfeld-Jimbo quantizations of type A_n^+ and B_n^+ are quadratic-linear Koszul algebras.

Квантизации Дринфельда-Джимбо как алгебры Кожуля

В.К. Харченко (Национальный автономный университет Мексики)

Алгебры Кожуля появляются во многих разделах современной математики: в алгебраической геометрии, в теории представлений, в некоммутативной геометрии, в топологии, в теории чисел, в теории псевдокорней некоммутативных многочленов. Мы покажем, что многопараметрические квантизации Дринфельда-Джимбо типа A_n^+ и B_n^+ являются алгебрами Кожуля относительно q-порождающих Вейля.

Rich groups, weal second-order logic and applications

Olga Kharlampovich (City University of New York, Graduate Center and Hunter College, USA)

We discuss first-order rich groups, i.e., groups where the first-order logic has the same power as the weak second order logic. Surprisingly, there are quite a lot of finitely generated rich groups, they are somewhere in between hyperbolic and nilpotent groups (these ones are not rich). We provide some methods to prove that groups (and other structures) are rich and describe some of their properties. As corollaries we look at Malcev's problems in various groups. These are joint results with A. Miasnikov and M. Sohrabi.

Associative universal enveloping algebras over an operad

Khoroshkin Anton (Higher School of economics, Russia)

With each given Lie algebra g one associates it's universal enveloping U(g) whose representations are in one-to-one correspondence with representations of g. The famous PBW theorem tells that there is a filtration on U(g) whose associative graded is isomorphic to the polynomial algebra. I will report on the generalizations of the notion of universal enveloping algebras and the PBW theorem for algebras of a different type, such as for commutative, Poisson, pre-Lie algebras, and explain a general criterion on PBW property for associative universal enveloping algebras of algebras over a given quadratic operad, based on the theory of Gröbner-Shirshov bases for operads.

On solutions of (weak) conformal classic Jang-Baxter equation on $\mathrm{Cur}(\mathrm{sl}_2(\mathbb{C}))$

Kozlov R. A.

Sobolev institute of mathematics; NSU

Motivated by the study of Lie bialgebras, a theory of Lie conformal bialgebra was established by J. Liberati (see [6]). It was introduced that there is one-to-one corresondence between the class of finite Lie conformal bialgebras which is free as a $\mathbb{C}[\partial]$-modulem and conformal Manin triple associated to a non-degenerate symmetric invariant conformal bilinear form. The notion of conformal classical Yang-Baxter equation (abbreviate it CCYBE) was also introduced to construct (coboundary) Lie conformal bialgebras and hence as a byproduct, the conformal Drinfeld's double was constructed.

Later, Y. Hong and C. Bai (see [3]) studied CCYBE more thoroughly and obtain notable results as, for examle, strict connections between solutions of CCYBE and left-symmetric conformal algebras or $\mathcal{O}$-operators which, in turn, connected with Rota-Baxter oarators.

Exlicitly, the weak version of CCYBE is defined as follows:

$$a_[[r,r]] := a_(\sum ([a_{i\lambda} a_j] \otimes b_i \otimes b_j|_{\lambda=1\otimes\partial\otimes 1} - a_i \otimes [a_{j\lambda} b_i] \otimes b_j|_{\lambda=1\otimes 1\otimes\partial} - a_i \otimes a_j \otimes [b_{j\lambda} b_i]|_{1\otimes\partial\otimes 1}) = 0 \ (mod\ \partial^{\otimes 3}) \in L^{\otimes 3}, \quad \forall a \in L, \quad (1)$$

where L is a Lie conformal algebra, $r = \sum a_i \otimes b_i \in L \otimes L$ and $\partial^{\otimes 3} = \partial \otimes 1 \otimes 1 + 1 \otimes \partial \otimes 1 + 1 \otimes 1 \otimes \partial$. If $[[r,r]] = 0 \ (mod\ \partial^{\otimes 3})$ then it is called just CCYBE.

Such a tensor r that satisfy (1) is called a solution of (weak) CCYBE.

A solution r of CCYBE (or the weak one) is called L-invariant, L is a conformal Lie algebra, if the following equality holds:

$$a_\lambda (r + \tau(r))|_{\lambda=-\partial^{\otimes 2}} = 0, \ \forall a \in L,$$

where τ is a linear operator interchanging tensor components. Be the action of L removed and the equation still proves true then the solution is called skew-symmetric.

Both version of CCYBE will be considered and within quite natural approach we obtain that there are no solutions other then one extended from the non-conformal case.

References

[1] A. D'Andrea and V.G. Kac, Structure theory of finite conformal algebras, Selecta Math. (N.S.) (3) **4** (1998) 3, 377–418.

[2] V. Chari, A. Pressley, A guide to quantum groups. Cambridge University Press, Cambridge, 1994.

[3] Y. Hong, C. Bai, Conformal classical Yang-Baxter equation, S-equation and $\mathcal{O}$-operators, Lett. Math. Phys. **110** (2020) 885–909.

[4] P.S. Kolesnikov, Homogeneous averaging operators on simple finite conformal Lie algebras, J. Math. Phys. **56** (2015), 071702, 10 p.

[5] E.I. Konovalova, Double Lie algebras, PhD thesis, Samara State University, 2009. 189 p. (in Russian).

[6] J. Liberati, On conformal bialgebras, J. Algebra **319** (2008) 2295–2318.

[7] Yu Pan, Q. Liu, C. Bai, L. Guo, PostLie algebra structures on the Lie algebra $\mathrm{sl}(2,\mathbb{C})$, Electron. J. Linear Algebra **23** (2012) 180–197.

DIFFERENT ZERO-DIVISOR GRAPHS OF A FINITE RING

A. S. MONASTYREVA

The zero-divisor graph $\Gamma(R)$ of a ring R is the graph whose vertices are nonzero zero divisors of the ring R (one- and two-sided), and two different vertices x and y are joined by an edge iff $xy = 0$ or $yx = 0$ (Anderson, Livingston, 1999, commutative case; S.Redmond, 2002, noncommutative case). To describe the rings whose zero-divisor graph satisfies a certain condition it has become one of the directions of investigations in this area.

The geometric depiction of the zero-divisor graph is rather complicated even for rings of small order. Therefore, it is necessary to partition the vertex set of the graph into cosets so that the impression of the structure of the graph as a whole be preserved. Some method for solving this problem was proposed by S.B.Mulay (2002), S.Spiroff, C.Wickham (2011), E.V. Zhuravlev, A.S. Monastyreva (2020, noncommutative case).

For nilpotent rings, the notion of partially compressed zero-divisor graph was introduced (A.S. Monastyreva, 2021).

In the talk, we will discuss our results on different zero-divisor graphs of a finite ring.

Altai State University, Barnaul (Russia)
E-mail address: akuzmina1@yandex.ru

DG Lie algebras, simplicial groups and their 1-truncations

Jacob Mostovoy (CINVESTAV-IPN)

The n-truncation of a chain complex consists of its graded components of degrees at most n. Similarly, the n-truncation of a simplicial set consists of its simplices of dimension at most n. For example, the 0-truncation of a DG Lie algebra is a usual Lie algebra and the 0-truncation of a simplicial group is an ordinary group. The left adjoint of the n-truncation functor is known as the n-skeleton (at least in the case of simplicial sets and groups) and was first studied by Conduché. The 1-truncation of a DG Lie algebra is an (augmented) Leibniz algebra and the 1-truncation of a simplicial group is known as a pre-crossed module. I will describe the corresponding 1-skeleton functors and show how they lead to homology theories similar to rack homology. I will also describe how this point of view on Leibniz algebras leads to the answer to a question of Pirashvili.

RIGHT ALTERNATIVE JUST INFINITE ALGEBRAS

A. S. PANASENKO

Definition. An algebra A over a field F is called *just infinite* iff $\dim_F A = \infty$ and $\dim_F A/I < \infty$ for every nonzero ideal $I \trianglelefteq A$.

Usual examples of associative and commutative just infinite algebras are polynomial ring $F[x]$ and ring of formal power series $F[[x]]$. In the world of $\mathbb{N}$-graded algebras just infinite algebras are analogous of simple algebras [1].

It is known that alternative (or Jordan) just infinite algebras are prime [2],[3]. There are examples of solvable (and even metabelian) just infinite Lie algebras [4].

Definition. An algebra A is called *right alternative* if A satisfies the following identities:

$$xy^2 = (xy)y,$$
$$((xy)z)y = x((yz)y).$$

Theorem. *Any just infinite right alternative algebra is prime.*

Definition. A right alternative algebra A is called *(-1,1)-algebra* if A satisfies the following identity:

$$(xy)z + (yz)x + (zx)y = x(yz) + y(zx) + z(xy).$$

I.R.Hentzel proved that any finitely generated prime (-1,1)-algebra is associative [5]. So, we have

Corollary. *Any just infinite finitely generated (-1,1)-algebra is associative.*

References

[1] *Z. Reichstein, D. Rogalski, and J. J. Zhang.* Projectively simple rings, Adv. Math., 203, N 2 (2006), 365–407.

[2] *A.S.Panasenko,* Just infinite alternative algebras. Math. Notes, 98, N 5 (2015), 805–812.

[3] *V.N.Zhelyabin, A.S.Panasenko,* Nearly finite-dimensional Jordan algebras. Algebra and Logic, 57, N 5 (2018), 336–352.

[4] *A.Shalev, E.Zelmanov,* Narrow Lie algebras: a coclass theory and a characterization of the Witt algebra, J. Algebra, N 189 (1997), 294-331.

[5] *I.R.Hentzel,* Nil semi-prime (-1,1)-ring. J.Algebra, 30, N 1 (1974), 236–258.

Novosibirsk State University

Email address: a.panasenko@g.nsu.ru

The structure of varieties of solvable Jordan algebras.

A. V. Popov

The talk will describe some results in the study of varieties of solvable Jordan algebras.

An important role among such varieties is played by the varieties of Jordan algebras satisfying the identities $x^4 \equiv 0$ and $(x_1 y_1)(x_2 y_2)(x_3 y_3) \equiv 0$. These varieties turn out to be related to Lie algebra varieties. This allows us to define for an arbitrary variety of solvable Jordan algebras $\mathcal{V}$ its enveloping variety of Lie algebras $\mathcal{L}(\mathcal{V})$.

Many properties of the variety $\mathcal{V}$ are completely or partially determined by the properties of the corresponding variety $\mathcal{L}(\mathcal{V})$. Among such properties are T-primacy, finite basability, and the growth of varieties.

On simple pre-Lie algebras of matrix type

Alexander Pozhidaev (Sobolev Institute of Mathematics, Russia)

In this talk we discuss the simple pre-Lie algebras, we present some results on the structure of the finite-dimensional right-symmetric (pre-Lie) algebras A possessing the matrix subalgebra M_n, whose unit is one of A. We discuss the notion of Endomorph for an arbitrary algebra, also we present some types of irreducible finite-dimensional unital right-symmetric bimodules over M_2.

Semialgebra systems

Loius Rowen (Bar Ilan University, Israel)

In tropical mathematics, as well as other mathematical theories involving semirings, one often is challenged by the lack of negation when trying to formulate the tropical versions of classical algebraic concepts for which the negative is a crucial ingredient. Following ideas originating in work of Dress, Gaubert, and the Max-Plus group and pursued further by Akian, Gaubert, and Guterman, we have coped with this difficulty by introduced algebraic structures with ``negation maps'', called systems, showing how these unify the more viable (super)tropical versions, as well as symmetrization, hypergroup theory and fuzzy rings, thereby helping to explain similarities in these theories. Special attention is paid to metatangible systems, whose algebraic theory includes the main examples, and is rich enough to facilitate computations and provide a host of structural results.

One has ``ground'' systems, insofar as they are the underlying structure which can be studied via other ``systemic modules,'' as in classical representation theory.

By formalizing the structure, one can introduce morphisms. Morphisms enable us to describe the tropicalization functor, as well as providing a link between classical algebraic results and their tropical and hyperfield analogs. In this framework we shall discuss linear algebra (joint work with Akian, Gaubert, Niv, and Sergeev), exterior semialgebras (joint work with Gatto), Lie semialgebras, Clifford semialgebras (joint work with Chapman and Gatto), and homology theory (joint work with Jaiung and Mincheva).

C_2-Graded groups, their Real representations and Dyson's tenfold way

Dmitri Rumynin (Warwick University, UK)

A C_2-graded group is a pair: a group G and its index two subgroup H. Its Real representation is a complex representation of H with an action of the other coset G\H of odd elements in another way that needs to be chosen. Different choices lead to different theories. Such representations appeared independently in three different disciplines: Algebra, Physics and Topology. The goal of the talk is to review the formalism and various choices, including resulting theories. The talk is based on my recent works with James Taylor (Oxford) and Matthew B. Young (Utah State).

Singular superalgebras

Oleg Shashkov, Sergei Pchelintsev (Financial University, Russia)

A simple right-alternative superalgebra in which the even part has zero multiplication is called *singular*. It is proved that every singular superalgebra with a finite-dimensional even part is an extended double. It is proved that there are no extended doubles of dimension 6, 7, 8, 11; constructed d-dimensional extended doubles for all other d≥5.

Some aspects of quasigroup theory

Shcherbacov Victor (Institute of Mathematics and Computer Science, Moldova)

We present some results obtained in the last twenty years in Belousov school (Chisinau, Moldova). It is possible to divide this talk on three parts: theoretical; applications of quasigroups in cryptology; quantitative estimates in binary groupoids

Multicomponent generalizations of MKDV equation and nonassociative algebraic structures

Shestakov Ivan (University of Sao Paolo),
joint work with V.V. Sokolov (Landau Institute for Theoretical Physics)

Relations between triple Jordan systems and integrable multi-component models of the modified Korteveg–de Vries type are established. The most general model is related to a pair consisting of a triple Jordan system and a skew-symmetric bilinear operation. If this operation is a Lie bracket, then we arrive at the Lie-Jordan algebra introduced earlier by A.Grishkov and I.Shestakov.

On braces, pre-Lie algebras and Hopf-Galois extensions

Smoktunowicz Agata (University of Edinburgh)

In 2014 Wolfgang Rump showed that there is a correspondence between braces and left nilpotent Pre-Lie algebras using methods from algebraic geometry ([1], pages 135, 136). In the case of finite braces and pre-Lie algebras Rump suggested using Lazard's correspondence ([1], page 141). In this talk we obtain simple formulas for the passage from braces to pre-Lie algebras and an elementary proof of 1-to-1 correspondence between strongly nilpotent F-braces and pre-Lie algebras over F (for F=Q and F=F_{p}.) As an application we obtain a simple formula for the inverse of the Baker-Cammpbel-Haustorf formula for adjoint groups of brace.

We also explain Rump's formula for the passage from pre-Lie algebras to braces. We mention some results on braces, Hopf-Galois extensions and pre-Lie algebras obtained using this correspondence between braces and pre-Lie algebras.

We also mention several open questions.

[1] W. Rump, The brace of a classical group, Note di Matematica, Volume 34, Issue 1 (2014).

ON THE LIE SOLVABILITY OF NOVIKOV ALGEBRAS

U.U. UMIRBAEV, V.N. ZHELYABIN, AND K.M. TULENBAEV

A nonassociative algebra N over a field F is called a *Novikov* algebra if it satisfies the following identities:

$$(xy)z - x(yz) = yx)z - y(xz)$$

$$(xy)z = (xz)y$$

I.M. Gelfand and I.Ya. Dorfman [1] firstly defined these identities in the study of Hamiltonian operators in the formal calculus of variations. These identities played an important role in the classification problem of linear Poisson brackets of hydrodynamical type, which was solved by A.A. Balinskii and S.P. Novikov [2]. It was shown in [4] for every n of the form $n = 2^k 3^l$ that a Z_n- graded Novikov algebra

$$N = N_0 \oplus \ldots \oplus N_{n-1}$$

over a field of characteristic not equal to $2, 3$ is solvable if N_0 is solvable. In [3] was prove that, if L is a right nilpotent subalgebra of a Novikov algebra N, then the right ideal of N generated by L^2 is right nilpotent.In [5] Z.Zhang and T.G.Nam proved that a Novikov algebra is Lie-nilpotent if and only if the ideal, generated by elements $ab - ba$ is nilpotent. We prove that this ideal is exactly $[N, N]$. This paper is devoted to the study of Lie-solvability of Novikov algebras.

Theorem. *Let N be a Lie-solvable Novikov algebra with solvability index n over field of F of characteristic $\neq 2$. Then the ideal $[N, N]$ is solvable.*

S.P. Mishchenko constructed an example of infinite-dimensional solvable Lie algebra with non nilpotent commutator in [6]. We use this approach and construct infinite-dimensional Novikov algebra L such that corresponding Lie algebra $L^{(-)}$ of L is solvable, but the ideal $[L, L]$ of L is not nilpotent.

References

[1] I. M. Gel'fand, I. Ya. Dorfman, Hamiltonian operators and algebraic structures related to them. (Russian) Funct. Anal. Appl. 13 (1979), no. 4, 3–30.

[2] I.M. Balinskii, S.P. Novikov, Poisson brackets of hydrodynamic type, Frobenius algebras and Lie algebras. (Russian) Dokl. Akad. Nauk SSSR 283 (1985), no. 5, 1036–1039.

[3] U. Umirbaev, V. Zhelyabin, On the Solvability of graded Novikov Algebras, International Journal of algebra and computation to apear.

[4] V. Zhelyabin, U. Umirbaev, On the Solvability of $\mathbb{Z}_3$-Graded Novikov Algebras. Symmetry 312(2) (2021), 13.

[5] Z. Zhang, T.G. Nam. Lie nilpotent Novikov algebras and Lie solvable Leavitt path algebras. arXiv:2012.11191 (2020), 18.

[6] S.P. Mishchenko, On solvable subvarities of the variety generated by the Witt algebra. Math. USSR-Sb, 64:2, (1989), 415–426.

Department of Mathematics, Wayne State University, Detroit, MI 48202, USA
E-mail address: umirbaev@wayne.edu

Institute of Mathematics of the SB of RAS, Novosibirsk, 630090, Russia
E-mail address: vicnic@math.nsc.ru

Department of Mathematics, Suleyman Demirel University, Kaskelen city, Abylaikan1/1, Almaty, 040900, Kazakhstan
E-mail address: kaisar.tulenbayev@sdu.edu.kz

The lattice of varieties of associative algebras over $\mathbb{Q}$ vs the lattice of varieties of associative-commutative rings

Mikhail Volkov (Ural Federal University, Ekaterinburg, Russia)

The structure of non-commutative rings is significantly more sophisticated than that of commutative rings. Therefore our result may seem a bit surprising as it demonstrates that the lattice $\mathcal{A}$ of all varieties of associative algebras over the field $\mathbb{Q}$ of rational numbers is much easier than the lattice $\mathcal{C}$ of all varieties of associative-commutative rings: we construct a sort of a local embedding of the former into the latter. More precisely, the lattice $\mathcal{A}$ can be easily decomposed into a subdirect product of countably many intervals $\mathcal{A}_2, \mathcal{A}_3, \ldots, \mathcal{A}_n, \ldots$ such that the interval $\mathcal{A}_n$ is anti-isomorphic to the submodule lattice $L(P_n)$ of the $\mathbb{S}_n$-module P_n of all multilinear polynomials of the variables $x_1, \ldots, x_n$, and we show that for every finite partial sublattice L of $L(P_n)$, there exists a prime number p such that L is anti-isomorphic to a partial sublattice of the lattice of all varieties of associative-commutative rings of characteristic p. The constructed local embedding preserves certain lattice properties; in particular, it implies that the lattice $\mathcal{A}$ satisfies every lattice identity that holds in the lattice $\mathcal{C}$. On the other hand, we provide examples showing limitations of our approach; in particular, the embeddings of finite partial sublattices of the intervals $\mathcal{A}_n$ do not extend to finite partial sublattices of the lattice $\mathcal{A}$ as a whole.

The Superalgebras of Jordan Brackets Defined by the n-Dimensional Sphere

V. N. Zhelyabin, A. S. Zakharov

We study the generalized Leibniz brackets on the coordinate algebra of the n-dimensional sphere. In the case of the one-dimensional sphere, we show that each of these is a bracket of vector type. Each Jordan bracket on the coordinate algebra of the two-dimensional sphere is a generalized Poisson bracket. We equip the coordinate algebra of a sphere of odd dimension with a Jordan bracket whose Kantor double is a simple Jordan superalgebra. Using such superalgebras, we provide some examples of the simple abelian Jordan superalgebras whose odd part is a finitely generated projective module of rank 1 in an arbitrary number of generators. An analogous result holds for the Cartesian product of the sphere of even dimension and the affine line. In particular, in the case of the 2-dimensional sphere we obtain the exceptional Jordan superalgebra. The superalgebras we constructed give new examples of simple Jordan superalgebras.

Some new applications of Groebner-Shirshov bases theory

Zerui Zhang (South China Normal University)

We shall introduce some new applications of Groebner-Shirshov bases theory in various kind of algebras. More precisely, by applying Groebner-Shirshov bases theory, we construct a shuffle operad of Gelfand-Kirillov dimension strictly between 1 and 2; we offer fast algorithms for calculating the Gelfand-Kirillov dimension of finitely presented commutative algebras or bicommutative algebras; we show that the varieties of Lie-admissible algebras and Lie algebras form a PBW pair;and we construct free dibands (i.e. free idempotent dimonoids) and free tribands (i.e. free idempotent trioids) generated by an arbitrary set.

On the solvability of graded Novikov algebras

Zhelyabin V.N. (Sobolev Institute of Mathematics)

Let G be a finite additive abelian group. Then any G-graded Novikov algebra N over a field K with solvable 0-component is solvable if the characteristic of K does not divide the order of the group G. We also show that any Novikov algebra N with a finite solvable group of automorphisms G is solvable if its the algebra of invariants is solvable and the characteristic of K does not divide the order of the group G.

О СЖАТЫХ ГРАФАХ ДЕЛИТЕЛЕЙ НУЛЯ КОНЕЧНЫХ КОММУТАТИВНЫХ ЛОКАЛЬНЫХ КОЛЕЦ

Е.В. Журавлев, О.А. Филина

Пусть S – коммутативная полугруппа с нулем. Введем на S отношение эквивалентности: $\forall x, y \in S \;\; x \sim y \Leftrightarrow \mathrm{Ann}(x) = \mathrm{Ann}(y)$. Класс эквивалентности элемента $x \in S$ обозначим $[x]$, а соответствующее фактормножество $S/\!\sim$. Рассмотрим $S/\!\sim$ как полугруппу относительно операции $[x][y] = [xy]$. Сжатым графом делителей нуля $\Gamma(S/\!\sim)$ полугруппы S будем называть граф, вершинами которого являются элементы $S/\!\sim$ и две вершины $[x]$, $[y]$ (не обязательно различные) соединяются ребром (или петлей) тогда и только тогда, когда $[x][y] = [0]$ (равносильно $xy = 0$).

Пусть R – конечное коммутативное локальное кольцо с единицей, $J = J(R)$ – радикал Джекобсона, $F = R/J = GF(p^r)$. Существуют элементы $m_1, \ldots, m_h \in J$ такие, что кольцо R раскладывается в прямую сумму F-модулей (см. [1]):

$$R = F \oplus Fm_1 \oplus \ldots \oplus Fm_h.$$

Рассмотрим случай, когда $\mathrm{char} R = p$ и $\dim_F J/J^2 = 3$, $\dim_F J^2/J^3 = 1$, $\dim_F J^3 = 1$, $J^4 = 0$,

$$R = F \oplus Fu_1 \oplus Fu_2 \oplus Fu_3 \oplus Fv \oplus Fw,$$

где $\{u_1, u_2, u_3, v, w\}$ – базис J над полем F, $u_1, u_2, u_3 \in J \setminus J^2$, $v \in J^2 \setminus J^3$, $w \in J^3$. В работе [2] классифицированы с точностью до изоморфизма все кольца R указанного типа. В настоящее время нами построены графы делителей нуля $\Gamma(R/\!\sim)$ всех таких колец. Далее, для примера, рассмотрим одно из колец со следующим умножением базисных элементов: $u_1^2 = v$, $u_2^2 = w$, $u_3^2 = w$, $u_1 v = w$, (см. [2], теорема 3, пункт 8).

В этом случае

$$R = [1] \bigcup_{s_i, l_j \in F} [u_1 + s_i u_2 + l_j u_3] \bigcup_{n_i, k_j \in F} [u_2 + n_i u_3 + k_j v] \bigcup_{m_i \in F} [u_3 + m_i v] \cup [v] \cup [w] \cup [0],$$

где

$$[u_1 + s_i u_2 + l_j u_3] = F^*(u_1 + s_i u_2 + l_j u_3) + Fv + Fw,$$

$$[u_2 + n_i u_3 + k_j v] = F^*(u_2 + n_i u_3 + k_j v) + Fw,$$

$$[u_3 + m_i v] = F^*(u_3 + m_i v) + Fw,$$

$$[v] = F^* v + Fw, \;\; [w] = F^* w, \;\; [0] = \{0\}, [1] = R^*,$$

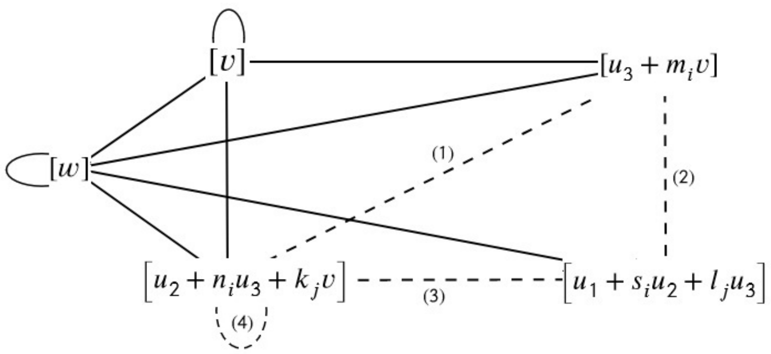
и для любых $s_i, l_j, n_i, k_j, m_i \in F$, $i, j \in \{1, \ldots, p^r\}$ справедливо:

$$\mathrm{Ann}[u_1 + s_i u_2 + l_j u_3] = \bigcup_{n_\alpha \in F} [u_2 + n_\alpha u_3 - (s_i + l_j n_\alpha) v] \cup [u_3 - l_j v] \cup [w] \cup [0],$$

$$\mathrm{Ann}[u_2 + k_j v] = \bigcup_{l_\beta \in F} [u_1 - k_j u_2 + l_\beta u_3] \bigcup_{m_\alpha \in F} [u_3 + m_\alpha v] \cup [v] \cup [w] \cup [0],$$

$$\mathrm{Ann}[u_2+n_iu_3+k_jv]=\bigcup_{l_\beta\in F}[u_1-(k_j+l_\beta n_i)u_2+l_\beta u_3]$$
$$\bigcup_{k_\alpha\in F}\left[u_2-\frac{1}{n_i}u_3+k_\alpha v\right]\cup[v]\cup[w]\cup[0], n_i\neq 0,$$
$$\mathrm{Ann}[u_3+m_iv]=\bigcup_{s_\alpha\in F}[u_1+s_\alpha u_2-m_iu_3]\bigcup_{k_\alpha\in F}[u_2+k_\alpha v]\cup[v]\cup[w]\cup[0],$$
$$\mathrm{Ann}[v]=\bigcup_{n_\alpha,k_\beta\in F}[u_2+n_\alpha u_3+k_\beta v]\bigcup_{m_\alpha\in F}[u_3+m_\alpha v]\cup[v]\cup[w]\cup[0],$$
$$\mathrm{Ann}[w]=J.$$

На рисунке 1 представлено геометрическое изображение графа $\Gamma(R/\sim)$, за исключением вершин [0] и [1] ([0] смежна со всеми вершинами, а [1] смежна только [0]).



1) если $n_i=0$;
2) если $m_i+l_j=0$;
3) если $k_j+s_\alpha+l_\beta n_i=0$;
4) если $n_in_j+1=0$.

рис. 1

В данном изображении вершины $[u_1+s_iu_2+l_ju_3]$, $[u_2+n_iu_3+k_jv]$, $[u_3+m_iv]$ это группы вершин графа $\Gamma(R/\sim)$, причем пунктирные ребра означают смежность вершин графа при выполнение некоторых условий, указанных внизу рисунка.

Данная работа продолжает исследования по построению графов делителей нуля коммутативных колец порядка p^{6r} (для колец порядка p^{5r} задача решена в [3]). Этот результат, как пример, важен для актуальной в настоящее время тематике по классификации конечных колец, удовлетворяющих некоторому условию на их графы делителей нуля.

Список литературы

[1] R. Raghavendran, *Finite associative rings*, Compositio Math., **21** (1969), 195–229.
[2] E.V. Zhuravlev, *On the classification of finite commutative local rings*, Siberian Electronic Mathematical Reports, **3** (2006), 15–29.
[3] N. Bloomfield, *The zero divisor graphs of commutative local rings of order p^4 and p^3*, Communication in Algebra, **41** (2013), 765–775.

Алтайский государственный университет, г. Барнаул
E-mail address: evzhuravlev@mail.ru, olya-filina@mail.ru