A Variable Neighborhood Search Based Matheuristic for the Drilling Rig Routing Problem

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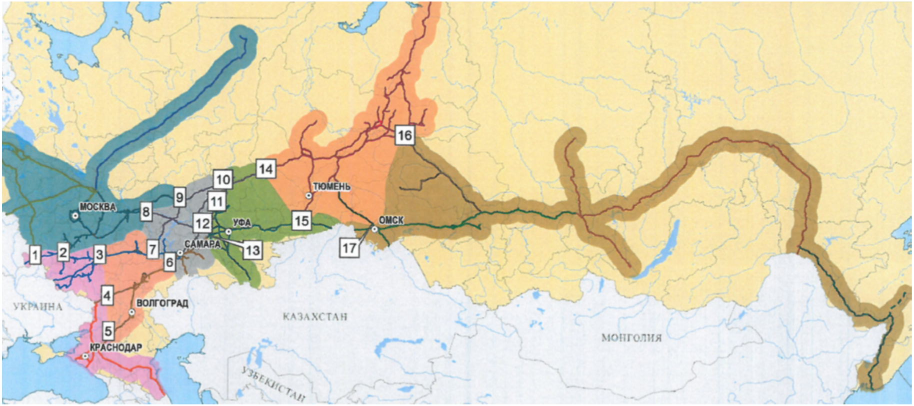
Drilling Map



Problem Definition

Features:

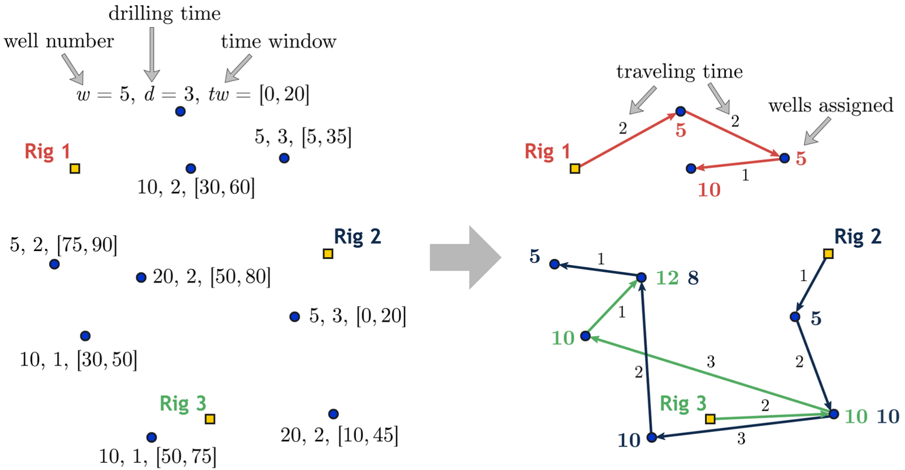
- well number
- time windows
- splittable service time
- work on a well cannot be split
- different rig types

Drilling Rigs

Exploration Sites

The objective is to minimize the total traveling cost, while respecting the time-window constraints

Example



total traveling time = 20

Notation

- K is the heterogeneous set of vehicles (drilling rigs),
- $\mathcal{V}^{\text{DEP}} = \{v_1, v_2, \ldots, v_{|K|}\}$ is the set of vehicle depots,
- $\mathcal{V}^{\text{CST}} = \{v_{|K|+1}, v_{|K|+2}, \ldots, v_{|K|+n}\}$ is the set of customers (exploration sites),
- $\mathcal{V} = \mathcal{V}^{\text{CST}} \cup \mathcal{V}^{\text{DEP}}$,
- $\mathcal{V}_k = \mathcal{V}^{\text{CST}} \cup \{v_k\}$,
- w_i is the number of wells for customer $i \in \mathcal{V}^{\text{CST}}$,
- $[e_i, l_i]$ is the time window of customer $i \in \mathcal{V}^{\text{CST}}$,
- d_{ik} is the time to drill a well for customer i by vehicle k,
- t_{ijk} is the traveling time between $i, j \in \mathcal{V}$ for vehicle k.

Variables

$$x_{ijk} = \begin{cases} 1, & \text{if vehicle } k \in K \text{ traverses arc } (i,j) \in \mathcal{V}_k^2, \\ 0, & \text{otherwise;} \end{cases}$$

$$y_{ik} = \begin{cases} 1, & \text{if vehicle } k \in K \text{ visits vertex } i \in \mathcal{V}_k, \\ 0, & \text{otherwise.} \end{cases}$$

Variables s_{ik}, $i \in \mathcal{V}_k$, $k \in K$, set service starting times for customer i and vehicle k.

Variables $w_{ik} \in \mathbb{Z}_{\geq 0}$, $i \in \mathcal{V}_k$, $k \in K$, set the number of wells vehicle k should drill for the customer i.

Mathematical Model

$$\min \sum_{k\in K} \sum_{i\in\mathcal{V}_k} \sum_{j\in\mathcal{V}_k} t_{ijk}x_{ijk} \qquad \text{traveling distance} \tag{1}$$

$$y_{ik} = \begin{cases} 1, & v_i = v_k, \\ 0, & v_i \neq v_k, \end{cases} \quad i \in \mathcal{V}^{\mathrm{DEP}}, k \in K, \tag{2}$$

depot variables

$$s_{v_k,k} = 0, \;\; w_{v_k,k} = 0, \;\; k \in K, \tag{3}$$

$$\sum\nolimits_{j\in\mathcal{V}_k} x_{ijk} = \sum\nolimits_{j\in\mathcal{V}_k} x_{jik} = y_{ik}, \;\; i \in \mathcal{V}_k, k \in K, \qquad \text{path flow} \tag{4}$$

$$\sum\nolimits_{k\in K} w_{ik} = w_i, \;\; i \in \mathcal{V}^{\mathrm{CST}}, \qquad \text{all wells are assigned} \tag{5}$$

$$w_{ik} \geq y_{ik}, \;\; i \in \mathcal{V}^{\mathrm{CST}}, k \in K, \qquad \text{drill at least one well if visiting} \tag{6}$$

$$w_{ik} \leq w_i y_{ik}, \;\; i \in \mathcal{V}^{\mathrm{CST}}, k \in K, \quad \text{can't drill if not visiting} \tag{7}$$

$$s_{ik} \geq e_i, \;\; s_{ik} + d_{ik}w_{ik} \leq l_i, \;\; i \in \mathcal{V}^{\mathrm{CST}}, k \in K, \qquad \text{time windows} \tag{8}$$

$$s_{ik} + d_{ik}w_{ik} + t_{ijk} - s_{jk} \leq \Delta_{ijk}(1 - x_{ijk}), \;\; i \in \mathcal{V}_k, j \in \mathcal{V}^{\mathrm{CST}}, k \in K, \tag{9}$$

$$x_{ijk},\; y_{ik} \in \{0,1\},\; s_{ik}, w_{ik} \in \mathbb{Z}_{\geq 0}, \;\; (i,j) \in \mathcal{V}_k^2, k \in K. \tag{10}$$

Penalties

$$L(x,\tau) = \min \sum_{k\in K} \sum_{i\in\mathcal{V}_k} \sum_{j\in\mathcal{V}_k} t_{ijk}x_{ijk} + \sum_{i\in\mathcal{V}^{\mathrm{CST}}} \gamma\tau^i_{\max} \qquad (11)$$

Additional constraints for new variables $\tau_{ik} \geq 0$, $\tau^i_{\max} \geq 0$ indicating tardiness for each pair (i,k), $i \in \mathcal{V}^{\mathrm{CST}}, k \in K$:

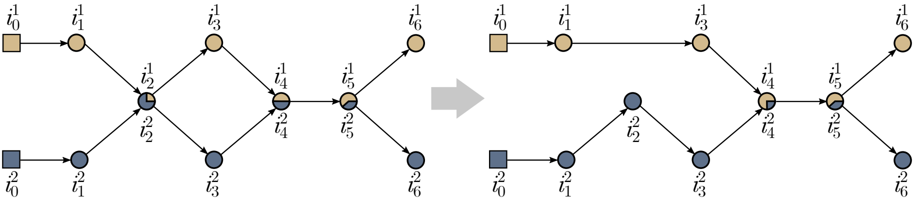
$$\tau_{ik} \geq s_{ik} + d_{ik}w_{ik} - l_i, \;\; i \in \mathcal{V}^{\mathrm{CST}}, k \in K, \qquad (12)$$

$$\tau^i_{\max} \geq \tau_{ik}, \;\; i \in \mathcal{V}^{\mathrm{CST}}, k \in K. \qquad (13)$$

The Work Repartition Subproblem

Provided: The routes represented as the total preorders $I_k = \{i_0^k, i_1^k, \ldots, i_{u_k}^k\}$, $i_l^k \preccurlyeq_{I_k} i_m^k$ if $l \leq m$, or $l = 0$, or $m = 0$.

Problem: To find new values for variables w_{ik}, $i \in \mathcal{V}^{\mathrm{CST}}$, $k \in K$, in order to improve the work partition.



Mathematical Model of the Subproblem

$$\min \sum_{k\in K}\sum_{i\in I_k}\sum_{j\in I_k, i\prec_{I_k} j} t_{ijk}x'_{ijk} + \sum_{i\in\mathcal{V}^{\mathrm{CST}}} \gamma\tau^i_{\max} \quad (14)$$

$$y'_{ik} \le y_{ik}, \quad i\in\mathcal{V}_k, k\in K. \quad (15)$$

$$\sum_{j\in I_k, i\prec_{I_k} j} x'_{ijk} = \sum_{j\in I_k, j\prec_{I_k} i} x'_{jik} = y'_{ik}, \quad i\in I_k, k\in K, \quad (16)$$

$$\sum\nolimits_{k\in K} w_{ik} = w_i, \quad i\in\mathcal{V}^{\mathrm{CST}}, \quad (17)$$

$$w_{ik} \ge y'_{ik}, \quad k\in K, i\in\mathcal{V}^{\mathrm{CST}}, \quad (18)$$

$$w_{ik} \le w_i y'_{ik}, \quad k\in K, i\in\mathcal{V}^{\mathrm{CST}}, \quad (19)$$

$$s_{ik} \ge e_i, \quad s_{ik} + d_{ik}w_{ik} \le l_i + \tau_{ik}, \quad k\in K, i\in I_k, \quad (20)$$

$$s_{ik} + d_{ik}w_{ik} + t_{ij} - s_{jk} \le M(1 - x'_{ijk}), \; i\in I_k, j\in I_k\setminus\{v_k\}, i\prec_{I_k} j, k\in K, \quad (21)$$

$$\tau_{ik} \ge s_{ik} + d_{ik}w_{ik} - l_i, \quad i\in I_k, k\in K, \quad (22)$$

$$\tau^i_{\max} \ge \tau_{ik}, \quad i\in I_k, k\in K, \quad (23)$$

$$x'_{ijk}\in\{0,1\}, \; s_{ik}\ge 0, \; \tau_{ik}\ge 0, \quad i,j\in I_k, i\prec_{I_k} j, k\in K, \quad (24)$$

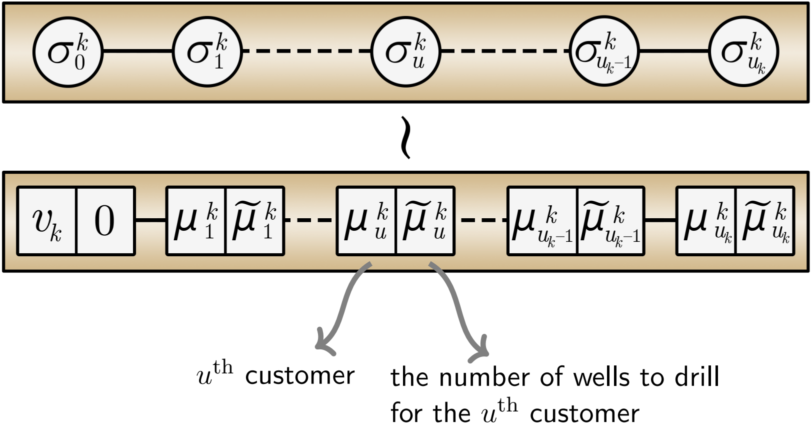
$$y'_{ik}\in\{0,1\}, \; w_{ik}\in\mathbb{Z}_{\ge 0}, \; \tau^i_{\max}\ge 0, \quad i\in\mathcal{V}_k, k\in K. \quad (25)$$

Initial Solution

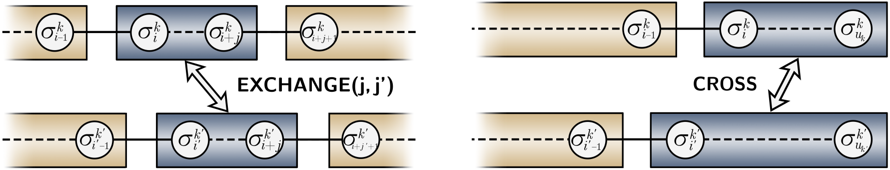
We use the greedy heuristic that consists of three steps:

- assign customers to the nearest vehicles
- generate a route permutation, using nearest-neighbor heuristic
- approximately solve repartition subproblem

Solution Representation



Route Neighborhoods

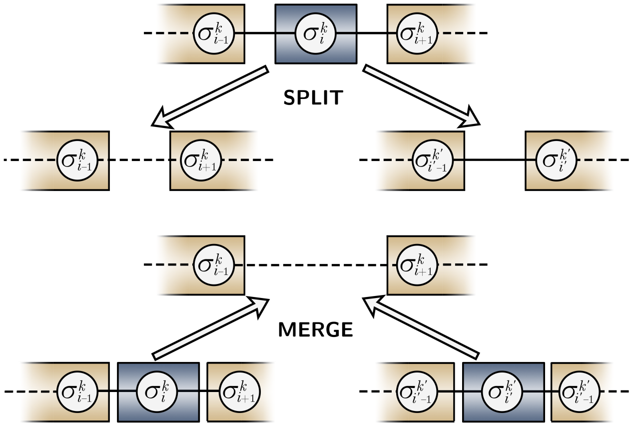


- Relocate: $j \in \{1, 2\}, j' = 0$
- Exchange: $j, j' \in \{1, 2\}$
- Cross-Exchange: $j \in \{1, \ldots, L\}, j' \in \{0, 1, \ldots, L\}$

To effectively handle time-window constraints, the subsequence structures from Vidal et al. (2013) [†] are used.

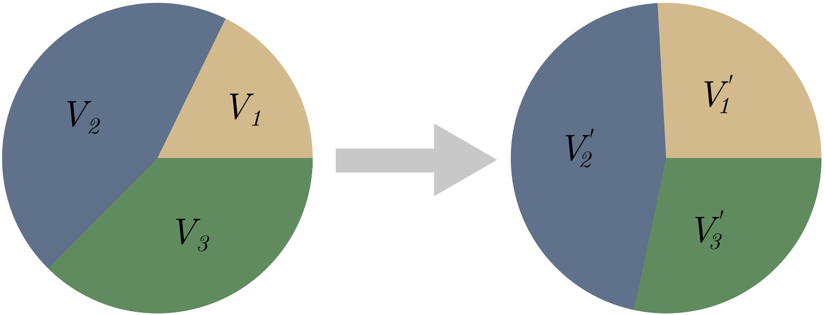
[†] Vidal T., Crainic T. G., Gendreau M., Prins C.: A hybrid genetic algorithm with adaptive diversity management for a large class of vehicle routing problems with time-windows. Comput. Oper. Res. **40**(1), 475–489 (2013).

Partition Neighborhoods



In addition, Kernighan–Lin neighborhood combining all the others is used in the local search.

Repartition Neighborhood



$$|V_i' - V_i| \leq R$$

Algorithm

Algorithm 1 General Variable Neighborhood Search

function GENERAL_VNS(σ, $k_{\max}$, $l_{\max}$, $\mathcal{N}$, N)
  while stopping criterion is not reached **do**
    $k \leftarrow 1$
    while $k \leq k_{\max}$ **do**
      $\sigma' \leftarrow$ `Shake`(σ, k, $\mathcal{N}$)
      $\sigma'' \leftarrow$ `VND`(σ', $l_{\max}$, N)
      `Neighborhood_change_sequential_SA`(σ, σ'', k)
  Solve to optimality the work repartition subproblem w.r.t. the routes of σ
  return best found solution

Test Instances

Three instance sets:

S1 $d_{ik} = 2 \;\; \forall i \in \mathcal{V}^{\mathrm{CST}},\, k \in K$

S2 $d_{ik} \in \{1, 2, 3\}, d_{ik} = d_{ik'} \;\; \forall k, k' \in K$

S3 three vehicle types that affect d_{ik}

And their variations:

$$\begin{array}{ccc} \mathsf{S} & & \mathsf{S'} \\ [e_i, l_i] & \Rightarrow & [e_i, \frac{e_i + l_i}{2}] \\ K & & K' = 2K \end{array}$$

- The algorithm is implemented in C++. All experiments are conducted on a computer with an AMD Ryzen 5 2600 3.4 GHz CPU and 16 GB of RAM.
- The algorithm parameters are tuned with SMAC.

Comparison with Gurobi, $|\mathcal{V}^{\text{CST}}| = 50$, $|K|$=6

#	Gurobi		LB	VNS, 90 sec		VNS, 270 sec		VNS, 810 sec	
	t	τ		Avg	Min	Avg	Min	Avg	Min
S1.1	**63**	**0**	0	**0**	**0**	**0**	**0**	**0**	**0**
S1.2	**74**	**0**	0	1.45	**0**	0.14	**0**	**0**	**0**
S1.3	**79**	**0**	−6.33	0.89	**0**	0.13	**0**	**0**	**0**
S1.4	**61**	**0**	−8.20	2.25	**0**	0.66	**0**	**0**	**0**
S1.5	**82**	**0**	−2.44	0.21	**0**	**0**	**0**	**0**	**0**
S1.6	**64**	**0**	0	1.61	**0**	0.63	**0**	**0**	**0**
S1.7	**66**	**0**	0	0.15	**0**	**0**	**0**	**0**	**0**
S2.1	87	0	−14.94	3.22	0	−0.11	**−1.15**	−0.8	**−1.15**
S2.2	**72**	**0**	0	1.57	**0**	0.28	**0**	**0**	**0**
S2.3	**88**	**1**	−4.08	4.29	1.02	2.65	1.02	0.82	0
S2.4	**73**	**5**	0	1.24	**0**	0.49	**0**	0.24	**0**
S2.5	**70**	**0**	0	0.33	**0**	**0**	**0**	**0**	**0**
S2.6	**61**	**0**	0	1.75	**0**	**0**	**0**	**0**	**0**
S2.7	**59**	**0**	0	1.14	**0**	**0**	**0**	**0**	**0**
S3.1	**61**	**0**	0	1.26	**0**	0.33	**0**	**0**	**0**
S3.2	**64**	**0**	0	4.69	**0**	0.31	**0**	0.78	**0**
S3.3	**66**	**0**	0	0.8	**0**	0.15	**0**	**0**	**0**
S3.4	**54**	**0**	0	4.26	**0**	2.59	**0**	0.37	**0**
S3.5	**66**	**0**	0	3.59	**0**	2.58	**0**	0.15	**0**
S3.6	**56**	**0**	0	2.73	**0**	1.07	**0**	0.36	**0**
S3.7	**57**	**0**	0	1.93	1.75	1.93	1.75	1.75	1.75
Avg	×		−1.71	1.87	0.13	0.66	0.08	0.17	0.03

#	Gurobi		LB	VNS, 90 sec		VNS, 270 sec		VNS, 810 sec	
	t	τ		Avg	Min	Avg	Min	Avg	Min
S1′.1	**122**	**0**	−6.56	6.53	4.1	4.02	1.64	1.31	**0**
S1′.2	**133**	**0**	−3.76	4.61	3.01	2.78	1.5	1.65	0.75
S1′.3	**147**	**0**	−5.44	6.71	2.72	2.86	0.68	1.56	0.68
S1′.4	**114**	**0**	−11.40	5.2	2.63	2.81	1.75	1.49	0.88
S1′.5	**153**	**0**	−11.11	3.12	0.65	1.11	0.65	0.65	**0**
S1′.6	**137**	**0**	−8.03	16.2	10.22	10.22	8.03	7.45	5.11
S1′.7	**146**	**0**	−7.53	6.39	4.11	5.07	2.05	3.77	1.37
S2′.1	**116**	**0**	−0.86	3.56	1.72	2.16	0.86	1.03	0.86
S2′.2	**125**	**2**	−2.76	10.12	7.59	7.45	6.21	5.38	4.14
S2′.3	**140**	**0**	−1.43	6.81	3.57	4	2.14	2.43	0.71
S2′.4	**119**	**3**	−1.34	5.79	2.68	2.55	0.67	1.81	0.67
S2′.5	**130**	**0**	−0.77	1.82	**0**	1	**0**	0.15	**0**
S2′.6	**129**	**0**	−1.55	12.53	6.2	6.43	3.88	4.34	2.33
S2′.7	**131**	**3**	−1.86	4.33	3.11	3.17	2.48	2.67	1.86
S3′.1	**60**	**0**	0	4.55	1.67	2.33	**0**	2	**0**
S3′.2	**66**	**0**	0	6.36	1.52	2.42	1.52	0.76	**0**
S3′.3	**78**	**0**	0	5.47	2.56	2.95	2.56	1.54	**0**
S3′.4	**57**	**0**	0	4.16	**0**	0.7	**0**	0.7	**0**
S3′.5	**70**	**0**	0	2.39	**0**	1.29	**0**	1	**0**
S3′.6	**74**	**0**	0	10	5.41	7.16	5.41	3.65	1.35
S3′.7	**71**	**0**	0	6.62	4.23	4.93	4.23	4.37	2.82
Avg	×		−3.07	6.35	3.22	3.69	2.2	2.37	1.12

Gurobi time limit is 3 hours for each instance.

Comparison with Gurobi, $|\mathcal{V}^{\mathrm{CST}}| = 150$, $|\mathcal{V}^{\mathrm{CST}}| = 500$

$|\mathcal{V}^{\mathrm{CST}}| = 150$, $|\mathcal{V}^{\mathrm{DEP}}| = 12$: $f(\text{Gurobi}) \gg f(\text{VNS})$,
$f(\text{Gurobi(Init. Sol.)}) \approx f(\text{VNS})$.

$|\mathcal{V}^{\mathrm{CST}}| = 500$, $|\mathcal{V}^{\mathrm{DEP}}| = 30$: $f(\text{Gurobi(Init. Sol.)}) \gg f(\text{VNS})$.

Alternative Schemes, $|\mathcal{V}^{\mathrm{CST}}| = 150$

#	Proposed			No MILP		Only Tabu	
	Avg		Min	Avg	Min	Avg	Min
	t	τ					
S1.1	**175.5**	**0**	−1.42	0.97	−0.28	7.41	5.98
S1.2	**193.6**	**3**	−1.16	2.37	−2.06	6.84	1.52
S1.3	**186.2**	**0**	−1.18	0.7	−0.64	4.14	2.04
S1.4	**198.9**	**0.5**	−6.33	3.58	−3.87	6.87	1.52
S1.5	**193.9**	**0**	−2.01	3.51	0.05	5.67	3.15
S1.6	**180.3**	**0**	−1.83	2.38	−0.72	4.99	3.16
S1.7	**170**	**0**	−1.76	1.18	−0.59	7.88	4.71
S2.1	**193.9**	**0**	−1.5	1.03	0.05	6.76	4.18
S2.2	**186.8**	**0.8**	−1.95	4.36	−1.44	4.36	2.67
S2.3	206.4	8	−3.28	**−0.49**	−1.54	2.62	0.21
S2.4	214.3	1.3	−8.49	**−2.16**	−8.05	2.07	−2.33
S2.5	**180.6**	**0**	−2.55	1.44	−0.33	6.7	4.65
S2.6	**221.2**	**4.7**	−3.43	3.24	−7.16	4.62	−1.19
S2.7	**219.5**	**0**	−2.51	3.51	−1.59	8.02	6.15
S3.1	**151.8**	**0**	−2.5	1.19	−1.84	11.46	8.7
S3.2	151.3	0	−1.52	**−0.93**	−2.84	13.75	8.39
S3.3	**150.5**	**0**	−1.66	0.4	−0.33	10.96	6.98
S3.4	**133.9**	**0**	−3.66	0.82	−2.91	22.26	16.5
S3.5	**146.6**	**0**	−1.77	0.41	−3.14	12.69	8.46
S3.6	144.6	0	−1.8	**−1.11**	−5.26	15.91	12.03
S3.7	**140.4**	**0**	−2.42	1.07	−1.71	16.81	10.4
Avg	×		−2.61	+1.31	−2.20	+8.7	+5.14

#	Proposed			No MILP		Only Tabu	
	Avg		Min	Avg	Min	Avg	Min
	t	τ					
S1′.1	367.9	1.6	−8.05	−1.07	−8.05	**−3.23**	−4.4
S1′.2	359.7	1.2	−5.03	**−0.43**	−4.76	−0.03	−1.53
S1′.3	**351.9**	**0.1**	−1.67	2.81	0.31	2.81	1.45
S1′.4	360.6	2.7	−5.83	0.57	−5.83	**−3.87**	−6.09
S1′.5	**357.5**	**0.2**	−2.64	3.89	0.14	2.53	0.7
S1′.6	369.6	5.6	−4.14	1.22	−1.79	**−4.58**	−7.66
S1′.7	**357.5**	**0.7**	−4.53	0.69	−3.16	1.23	−1.23
S2′.1	**314.4**	**0.9**	−5.69	3.12	−5.69	2.81	−1.67
S2′.2	311.3	5.9	−5.21	0	−5.48	**−1.67**	−7.1
S2′.3	**361.8**	**2.1**	−1.78	0.6	−1.78	3.5	−1.25
S2′.4	**301.4**	**0.9**	−5.61	2.03	−5.28	4.93	1.8
S2′.5	316.1	0	−2.25	**−0.35**	−2.88	8.07	5.66
S2′.6	319.4	4.4	−8.09	0.55	−3.96	**−3.52**	−7.54
S2′.7	317.8	0.9	−5.14	**−1.77**	−4.83	1.81	−0.55
S3′.1	**169.3**	**0**	−3.72	2.42	−1.36	14.35	11.05
S3′.2	**171.2**	**0**	−6.54	1.58	−0.12	23.6	15.65
S3′.3	**177.8**	**0**	−3.26	1.46	−0.45	8.94	5.17
S3′.4	**186.6**	**0.1**	−8.85	8.58	−4.58	12.42	3.94
S3′.5	**174.5**	**0**	−3.15	1.49	−0.29	12.72	8.88
S3′.6	**176.4**	**0**	−4.76	2.15	−0.23	14	8.28
S3′.7	**164.6**	**0**	−2.19	1.64	−2.79	16.04	9.36
Avg	×		−4.67	+1.48	−2.99	+5.37	+1.57

There are 10 runs for each instance with the a time limit of 900 sec.

Conclusion

- The new Vehicle Routing Problem is considered which is modeled as a MILP problem.
- The well-drilling work partition subproblem is introduced that turns out to be relatively easy to solve even for large-sized instances.
- We designed a matheuristic approach integrating MILP in the VNS algorithm.
- It is planned to consider the variant of the problem with uncertain drilling times.