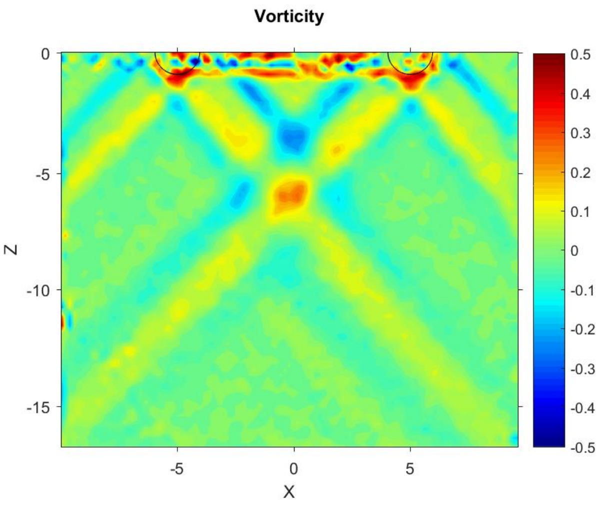
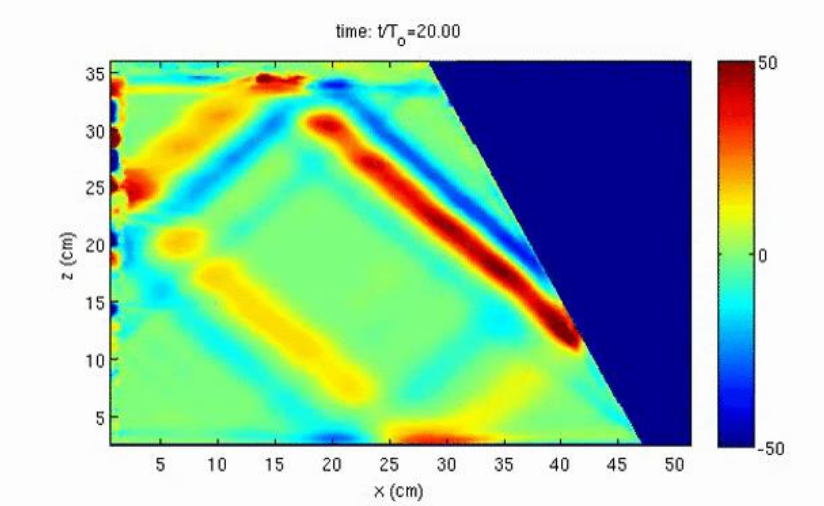
ФОКУСИРОВКА ВНУТРЕННИХ ВОЛН

<u>Шмакова Наталья</u>[1,2]

1. Институт гидродинамики им. М.А. Лаврентьева СО РАН, Новосибирск, Россия
2. Новосибирский государственный университет, Новосибирск, Россия





«Женщины в математике» 27-28 мая, Новосибирск

Внутренние волны в гео-
и астрофизических приложениях

Е. В. Ерманюк
З. В. Макридин
А. К. Хе
Н. Д. Шмакова
Е. В. Замараева
Я. Е. Рудая

Институт гидродинамики
им. М.А. Лаврентьева СО РАН
Новосибирск

И. Н. Сибгатуллин

Институт океанологии
им. П. П. Ширшова РАН
Москва

J.-B. Flor
B. Voisin
J. Sommeria

Laboratoire des
Écoulements
Géophysiques et
Industriels

С. В. Субботин
В. Г. Козлов

Пермский
государственный
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T. Dauxois
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P. Odier
H. Scolan
C. Brouzet
G. Pillet
S. Boury

Laboratoire de
Physique, ENS de
Lyon

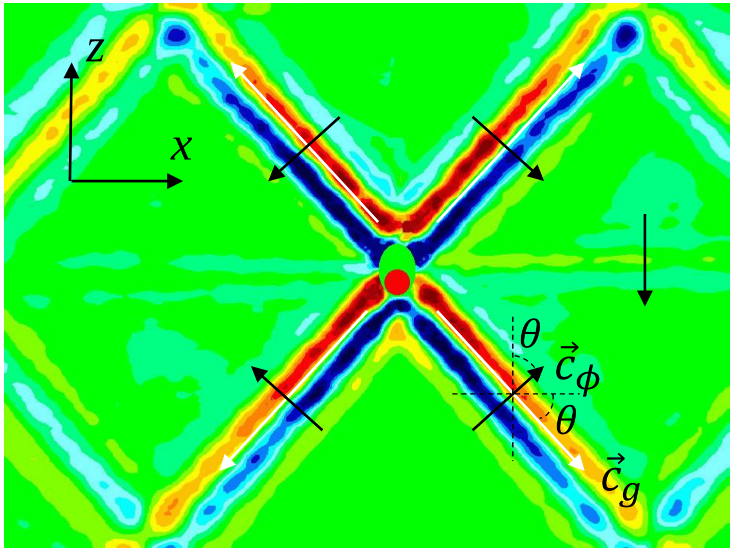
Дисперсионное соотношение

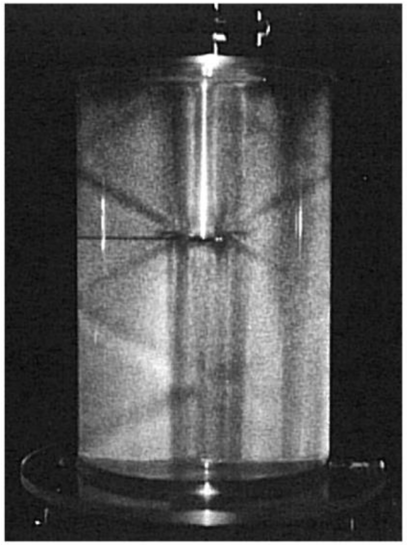
частота плавучести $N(z) = \left[-(g/\rho)d\rho/dz\right]^{1/2}$

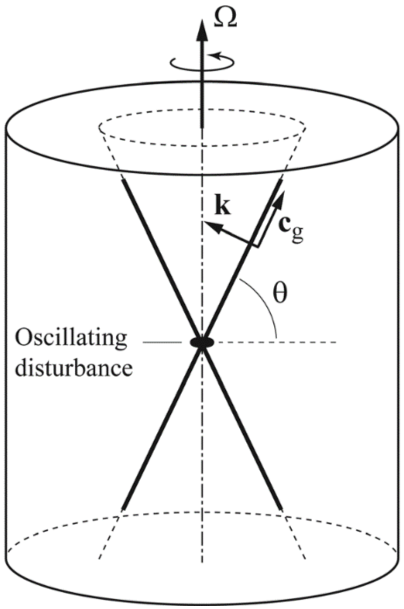
параметр Кориолиса 2Ω

$$\frac{\omega}{N} = \pm\sin\theta$$

$$\frac{\omega}{2\Omega} = \pm\cos\theta$$







внутренние волны в стратифицированной жидкости

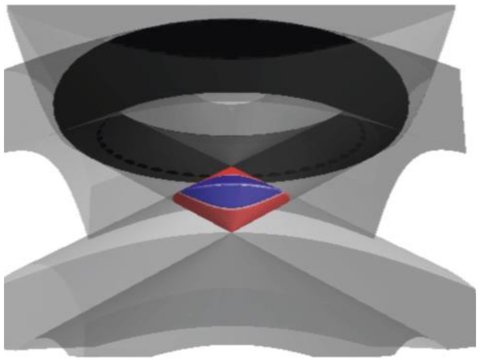
инерционные волны во вращающейся жидкости

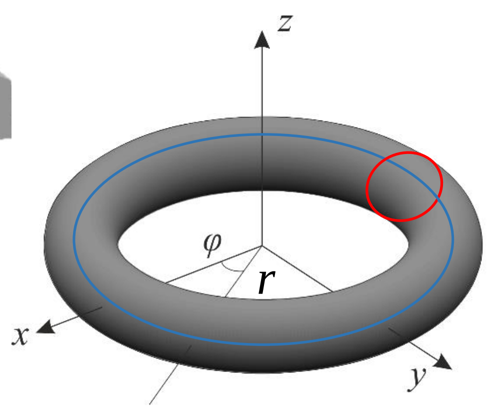
Фокусировка волн

«линза»

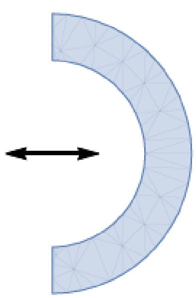
«бильярд»

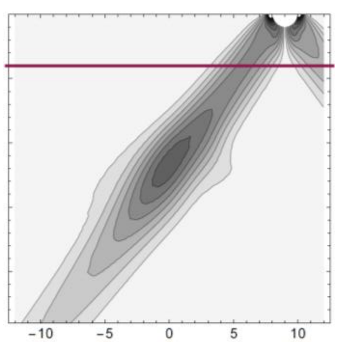
$$\frac{\omega}{N} = \pm \sin\theta$$

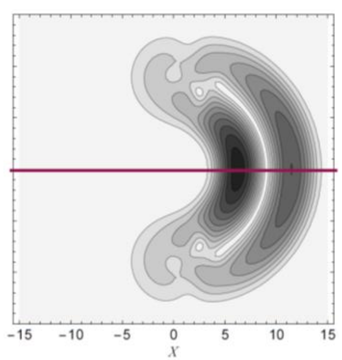




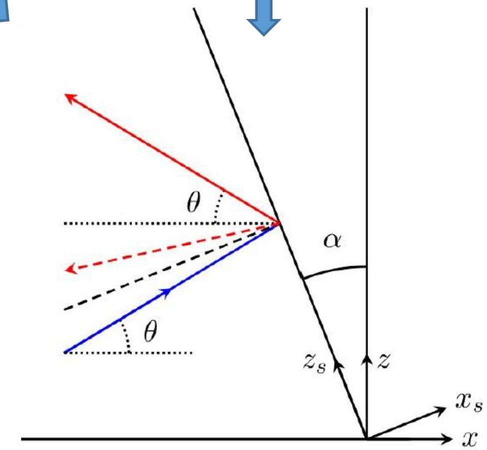
Фокусировка волн в зоне пересечения лучей от каждой секции тора



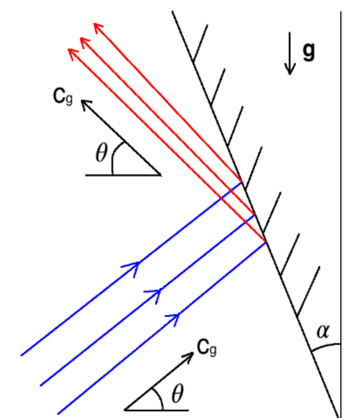




В случае незамкнутого объекта также фокусировка

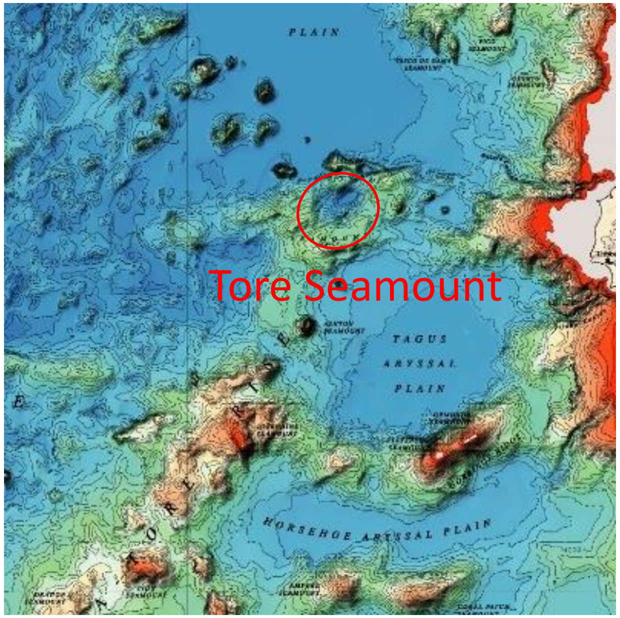


закон отражения



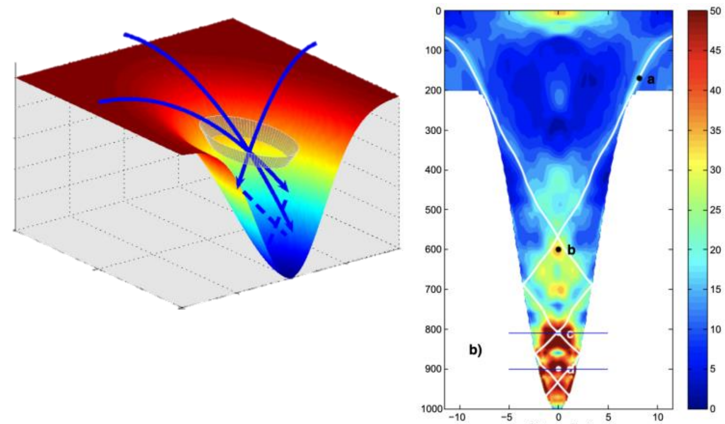
геометрическая фокусировка

Фокусировка волн горизонтальными колебаниями тора



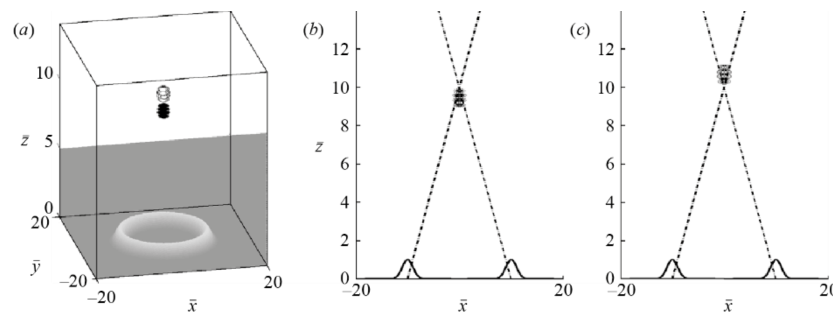
Peliz & Le Cann (2009)

- Tore Seamount as an example of meteorite crater with torus shape.
- The fluid column within the crater is well mixed possibly due to wave breaking in the zone of wave focusing



Vlasenko et al. (2016)

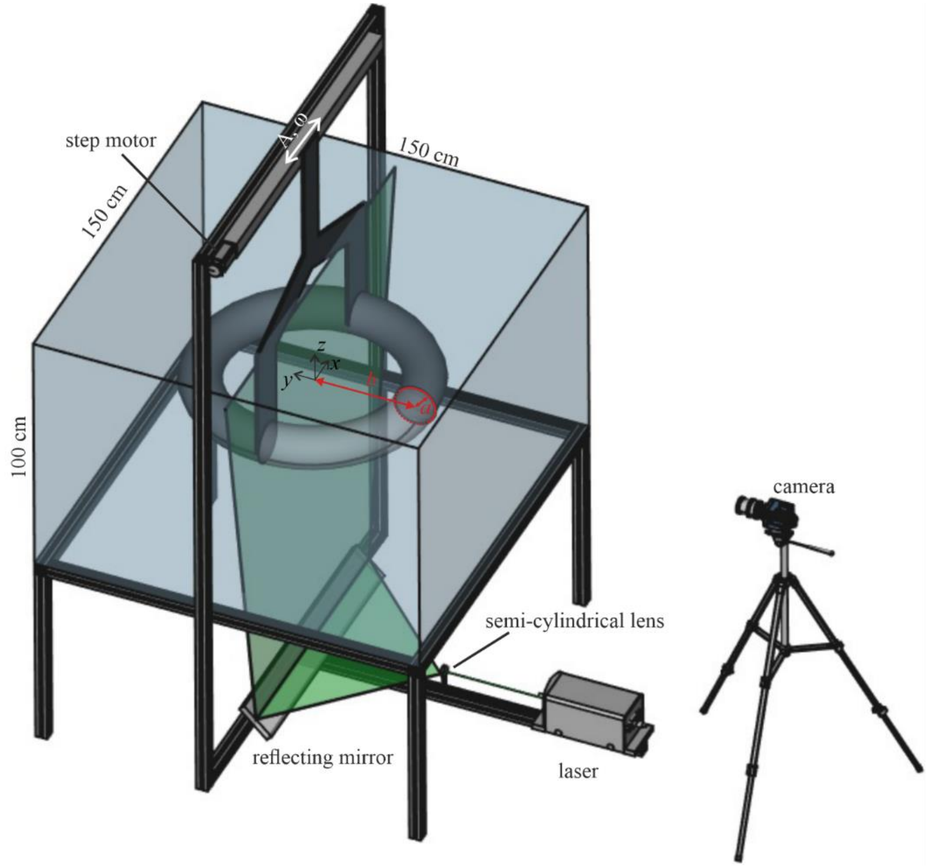
Amplification of the internal wave amplitude in fjords

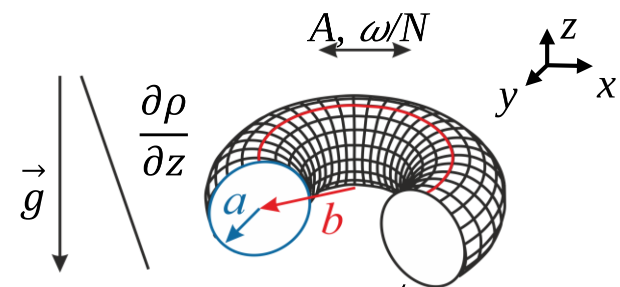


Bühler & Muller (2007)

Theoretical calculations of internal wave focusing – amplitude amplification in the focal zone

Фокусировка волн горизонтальными колебаниями тора





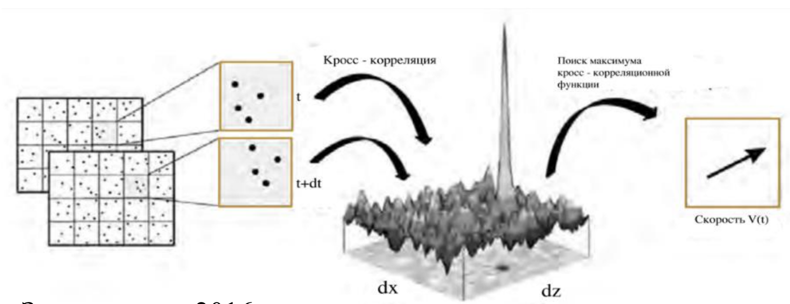
$$N = \left[-\frac{g}{\rho}\frac{\partial \rho}{\partial z}\right]^{1/2}$$

$$Ke = A/a$$

$$\varepsilon = b/a \quad St = \omega a^2/\nu$$

Torus	*a*, cm	*b*, cm	*St*
S	2	10	100
M	4	20	400
L	6	30	1000
XL	8	40	1800

visualization PIV, data treatment PIVLab (cross-correlation algorithm)



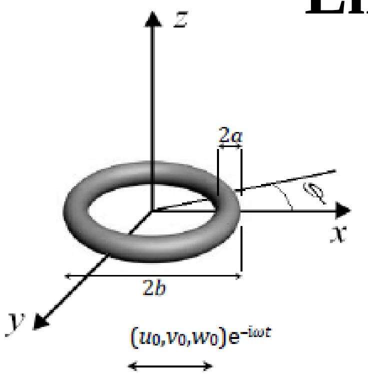
Захаров и др. 2016

$$C_{I1,I2}(m,n) = \sum_{l=-\frac{d_y}{2}}^{\frac{d_y}{2}} \sum_{k=-\frac{d_x}{2}}^{\frac{d_x}{2}} I_1(k,l) I_2(k+m, l+n)$$

W. Thielicke et al. 2021

Linear theory

Voisin (2011, 2016, corrections: 2020)



Oscillating body is represented as a source of mass releasing the volume

$$\nabla \cdot \boldsymbol{u} = q e^{-i\omega_0 t}$$

of fluid per unit volume per unit time, with $\boldsymbol{u}$ the velocity

The source satisfies $q(x,y,z) = 0$ for $|z| > a$, with body vertical size $2a$

$$(u,v,w) = \frac{\cos\theta}{8\pi^2} e_0^{-i\omega t} \int_0^{2\pi} d\phi_k [\sin\theta\cos(\phi_k - \phi), \sin\theta\sin(\phi_k - \phi), \cos\theta \ \operatorname{sgn}\ z]$$

$$\times \int_0^{\infty} dk\, k q(k_0, l_0, m_0) \exp\left(-\frac{\beta k^3 |z|}{\cos\theta}\right) \exp\{ik[r_h \cos\theta\cos(\phi_k - \phi) - |z|\sin\theta]\}$$

(r_h, ϕ, z) – cylindrical coordinates, $q(k,l,m) = \iiint q(x,y,z) e^{-i(kx, ly, mz)} dxdydz$ - source spectrum with wave vector components

$k_0 = \kappa\cos\theta\cos\phi_k$, $l_0 = \kappa\cos\theta\sin\phi_k$, $m_0 = -\kappa\sin\theta \operatorname{sgn} z + \frac{i\beta\kappa^3 \operatorname{sgnz}}{\cos\theta}$; $\beta = \frac{\nu}{2\omega_0\tan\theta}$, ν-kinematic viscosity

$$\epsilon = \frac{b}{a} \gg 1, Ke \ll 1, St \gg 1$$

Each section of the torus behaves as the section of a horizontal circular cylinder, with representation given in Voisin (JFM, 2020).

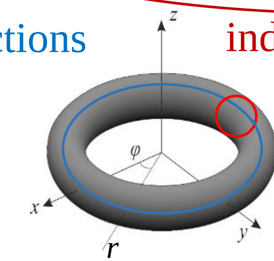
Using the wide separation between the local scale a of a single section and the global scale b of the whole torus we obtain the spectrum

$$q(k,l,m) = -4i\pi^2 NabAe^{i(\theta-\Theta)} J_0\left(\mathrm{b}\sqrt{k_2 + m_2}\right) J_1\left(a\sqrt{k^2 + l^2 + m^2}\right) k/\sqrt{k^2 + l^2 + m^2}$$

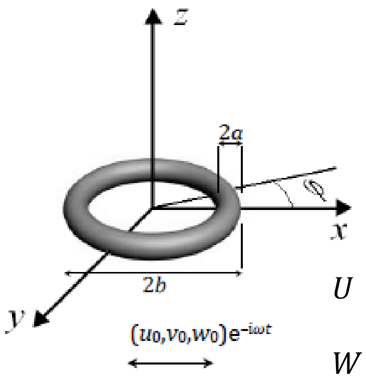
J_n - Bessel function of order n

the interface between sections

individual section



Теория



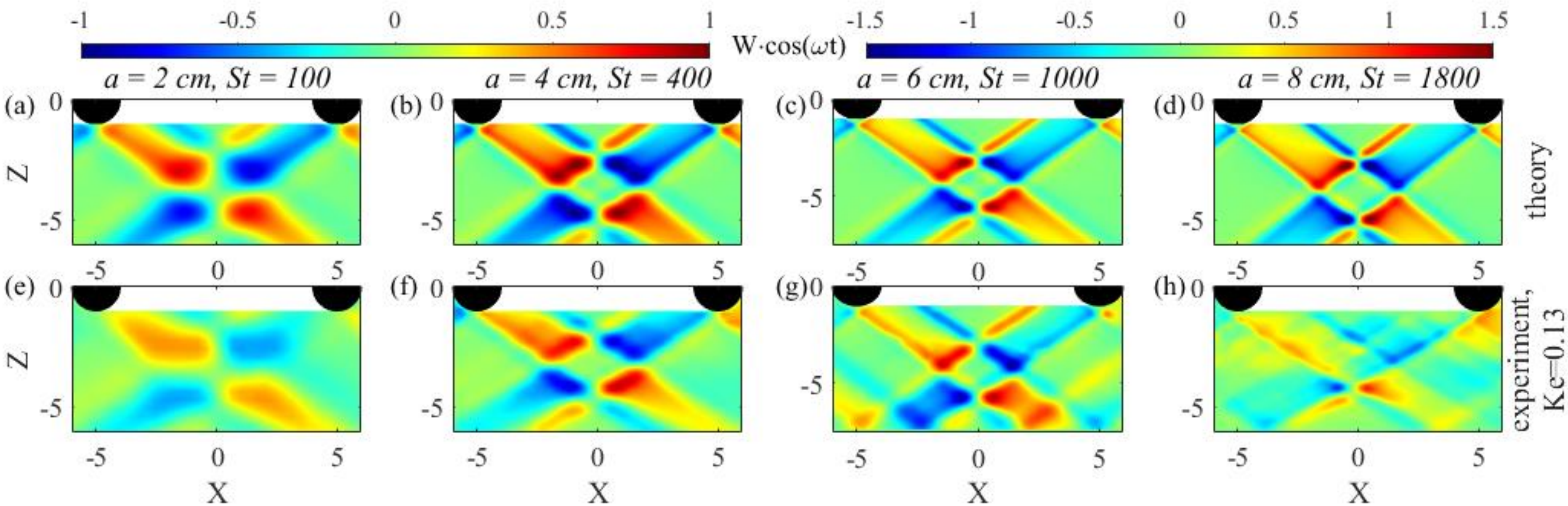
Theory: Voisin (2016); corrections: Shmakova et al. (2021)

$$(X,Y,Z) = (x,y,z)/a,\ (U,V,W) = (u,v,w)/(A\omega), T = \omega_0 t, \Theta = \pi/2$$

$$U = i\pi\epsilon\cos\theta\sin\theta e^{-i(T+\Theta-\theta)} \int_0^\infty K J_0(K\epsilon\cos\theta) J_1\left(K - i\frac{K^3}{2St}\right) e^{-\frac{K^3|Z|}{2St\sin\theta}} \times \left[J_2(KR\cos\theta)\frac{X^2}{R^2} - \frac{J_1(KR\cos\theta)}{KR\cos\theta}\right] e^{-iK|Z|\sin\theta} dK$$

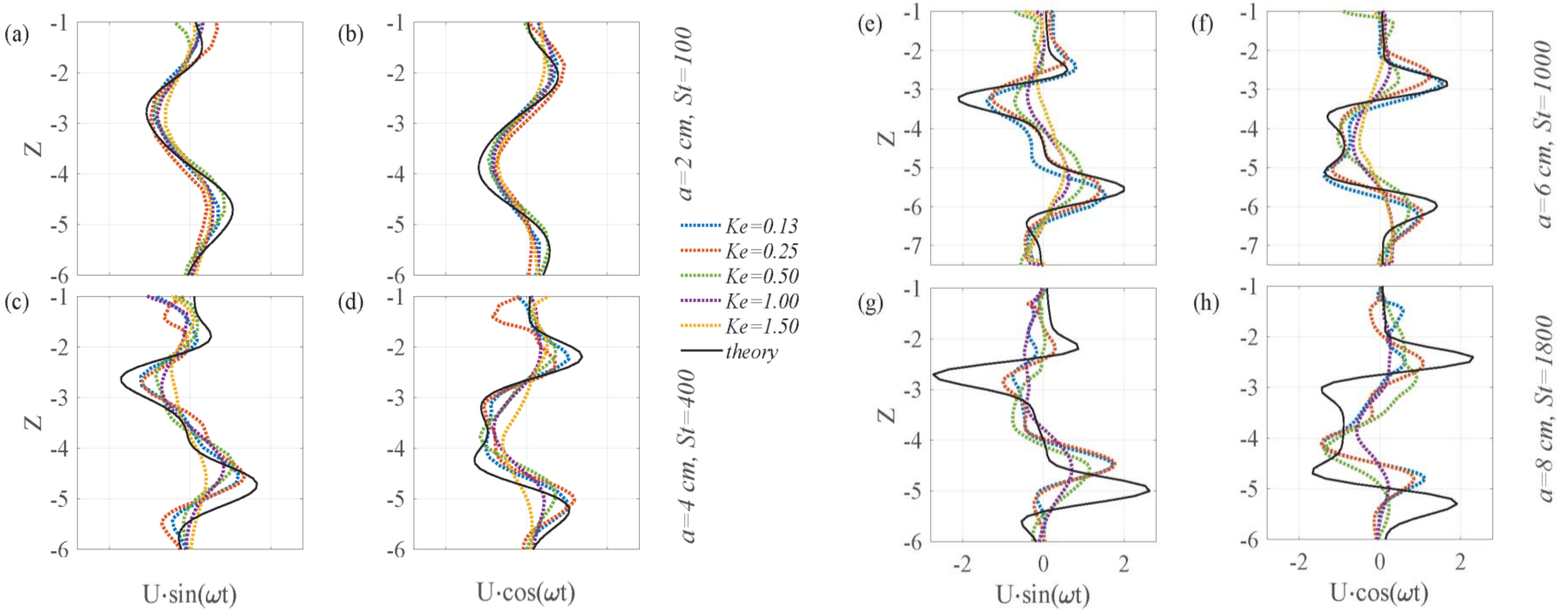
$$W = i\pi\epsilon\cos^2\theta e^{-i(T+\Theta-\theta)} \operatorname{sgn} Z \int_0^\infty K J_0(K\epsilon\cos\theta) J_1\left(K - i\frac{K^3}{2St}\right) e^{-\frac{K^3|Z|}{2St\sin\theta}} \times J_1(KR\cos\theta)\frac{X}{R} e^{-iK|Z|\sin\theta} dK$$

$$U \cdot \cos(\omega t) := \mathrm{Re}[\mathrm{U(t=0)}], \mathrm{W} \cdot \cos(\omega t) := \mathrm{Re}[\mathrm{W(t=0)}]$$



- Excellent qualitative comparison

Experimental and theoretical profiles along Z



- Excellent comparison for small St or small Ke
- For $St = 1800$ reflection at the surface perturbs wave focusing in the upper focal zone; nonlinear stratification – shift of the lower focal zone upwards with respect to the theoretical prediction
- Amplitude saturation for moderate and high St, possible explanation – triadic resonance instability, generation of wave turbulence

Energy spectra – to wave turbulence

2D PIV поле скорости $u(x,z,t)$ и $w(x,z,t)$

3D преобразование Фурье: $\hat{u}\,(x,z,t)$ и $\hat{w}(x,z,t)$

Спектр энергии:

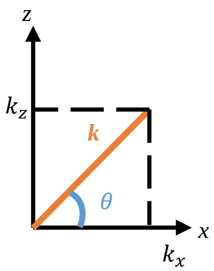
$$E\left(k_x,k_y,\omega\right)=\frac{|\hat{u}(k_x,k_z,\omega)|^2+|\hat{w}(k_x,k_z,\omega)|}{2ST}$$

interpolation

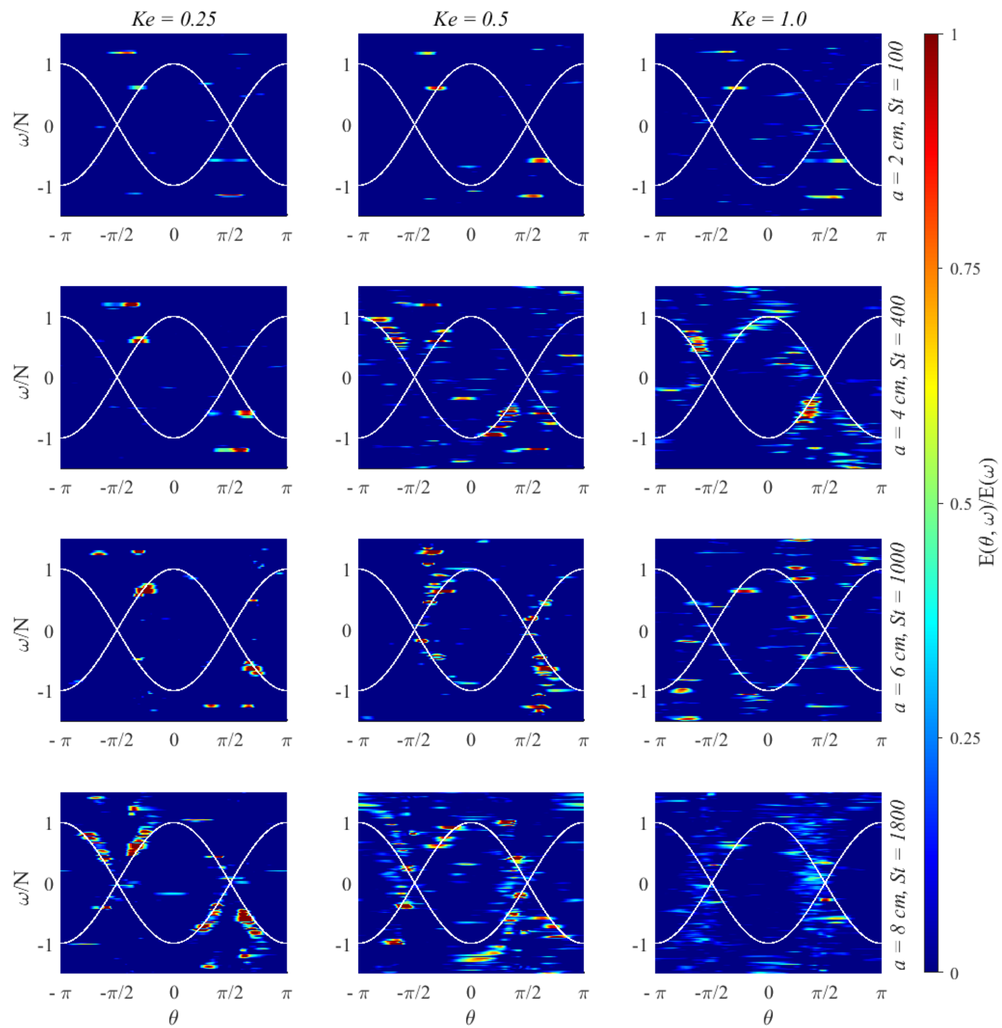
$$E(k,\theta,\omega)$$

$$\int_{k_{min}}^{k_{max}} dk$$

$$E(\theta,\omega)$$



Метод: Yarom & Sharon (Nature Physics 2014)

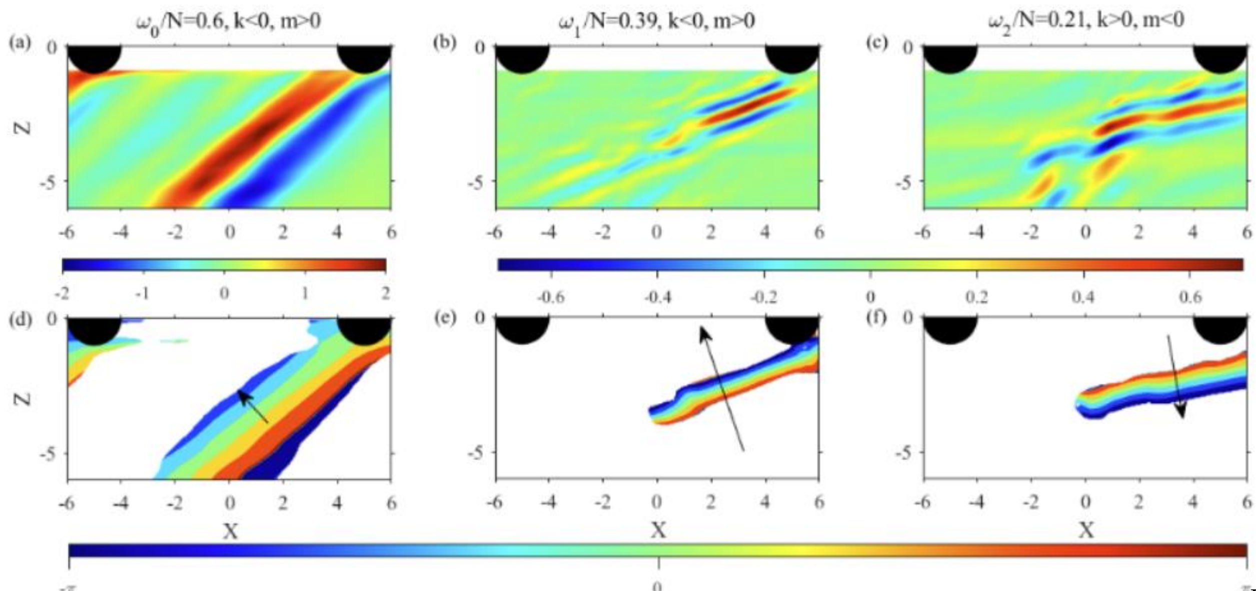


Ke \ St	0 … 0.2	0.2 … 0.5	0.5 … 0.8	≥ 0.8
100..250				TRI
400			TRI	TRI
1000		TRI	TRI	TRI
1800	TRI	TRI	TRI	TRI, WT
5800 (2021)	TRI	TRI	TRI	TRI

- With increase of Stokes number less energy needs to be introduced to the system to observe TRI
- At large St and Ke lots of internal waves at different frequencies (‘continuous’ spectrum) – wave turbulence

Triadic resonance

Фильтр по частотам и волновым векторам (Mercier et al. 2009)



Вертикальная компонента скорости

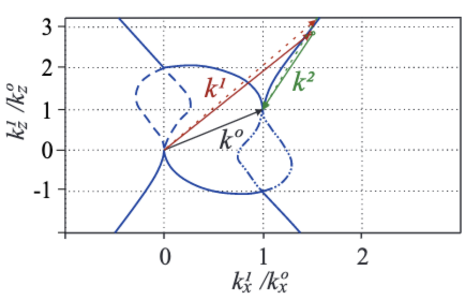
Фазовая скорость

Условия триадного резонанса: $\omega_0 = \omega_1 + \omega_2$; $\vec{k}_0 = \vec{k}_1 + \vec{k}_2$

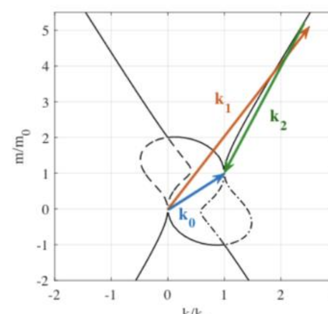
Into the dispersion relation $\frac{\omega}{N} = \sin\theta$

$$1 = s_1 \frac{|k_1/k_0|}{\sqrt{(k_1/k_0)^2(\omega_0/N)^2 + (m_1/m_0)^2[1-(\omega_0/N)^2]}} + s_2 \frac{|1-k_1/k_0|}{\sqrt{[1-(k_1/k_0)^2](\omega_0/N)^2 + [1-(m_1/m_0)^2][1-(\omega_0/N)^2]}}$$

Shmakova & Flor, 2019
Unimodal waves



$St = 100$
$Ke = 0.95$

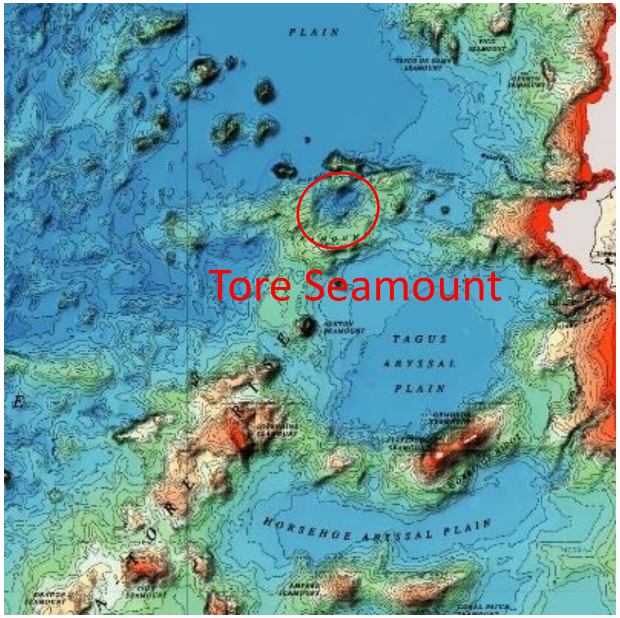


$St = 1000$
$Ke = 0.5$

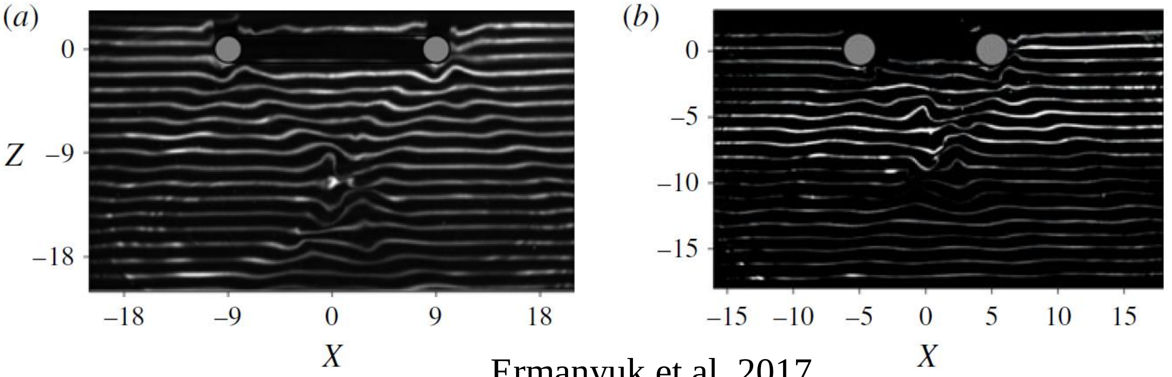
Triad for large Ke and St numbers: bimodal waves

- Triadic resonant instability for the first time in 3d Shmakova & Flor, JFM 2019

With the ‘torus’ geometry

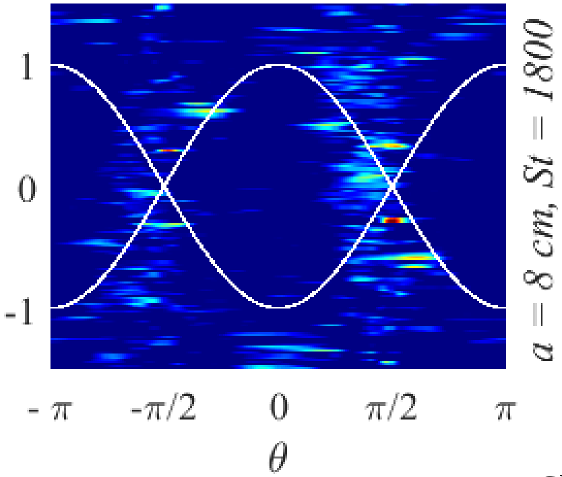


➤ Kelvin–Helmholtz instability in the focal zone – quick overturning at low St



Ermanyuk et al. 2017

➤ Triadic resonant instability to wave turbulence – slow mixing at large St



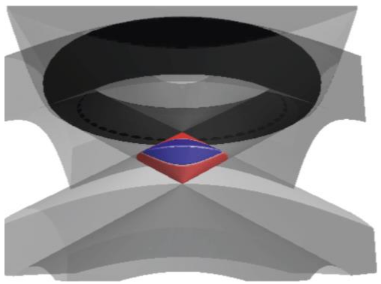
Shmakova et al. 2023 (prep)

Фокусировка волн

«линза»

«бильярд»

$$\frac{\omega}{N} = \pm\sin\theta$$



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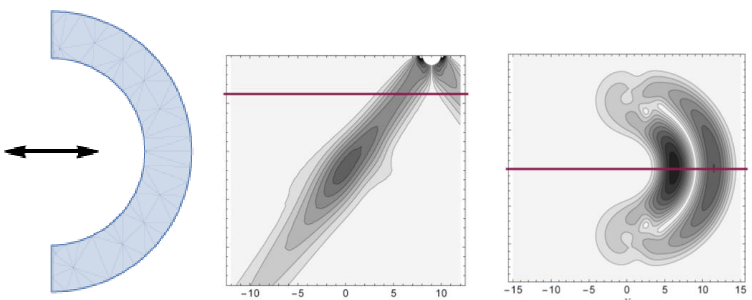
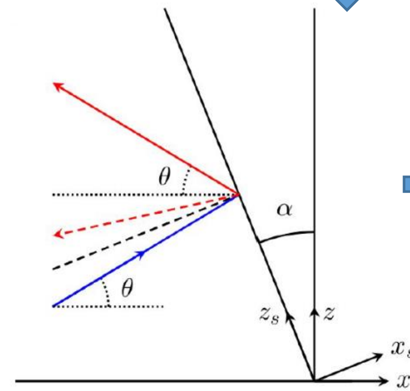
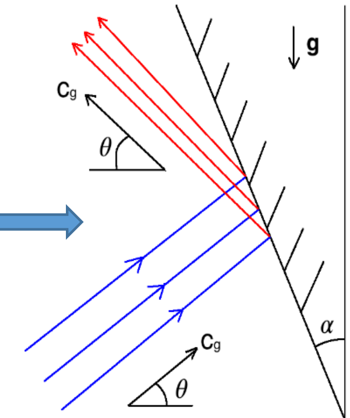


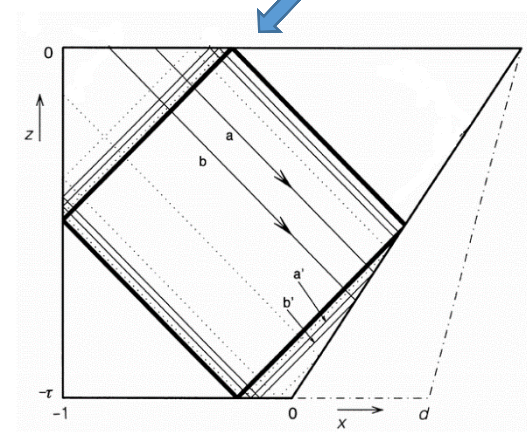
В случае незамкнутого объекта также фокусировка



закон отражения

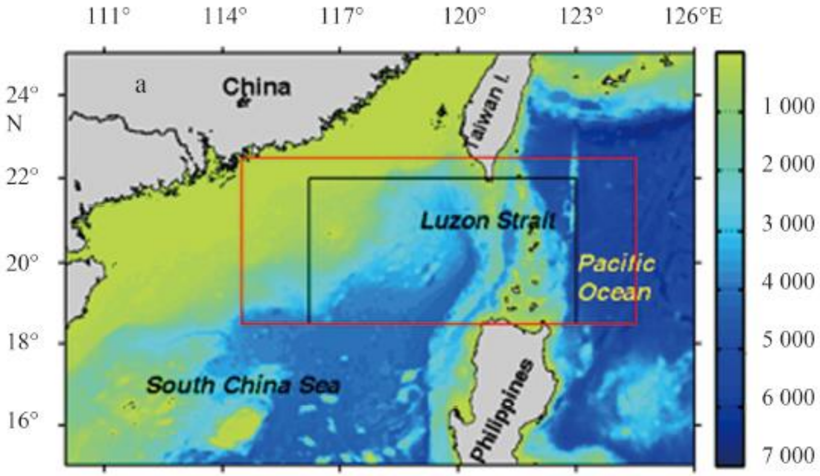


геометрическая фокусировка

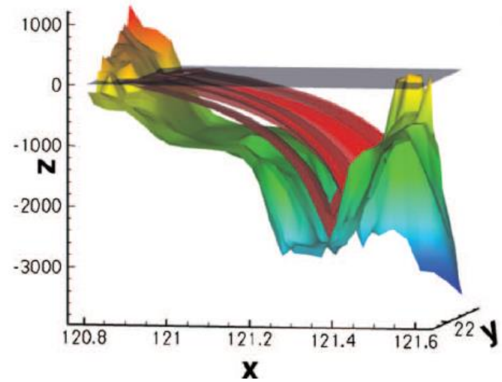


бильярд волновых лучей

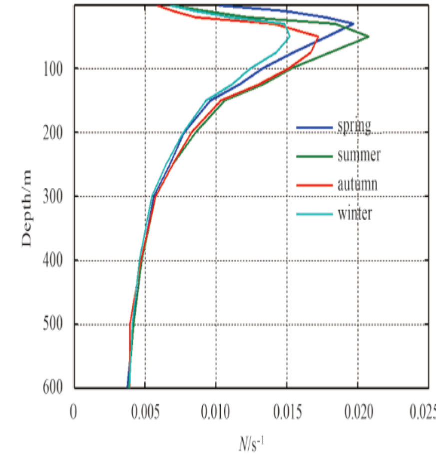
Геофизическое приложение: стратифицированная жидкость



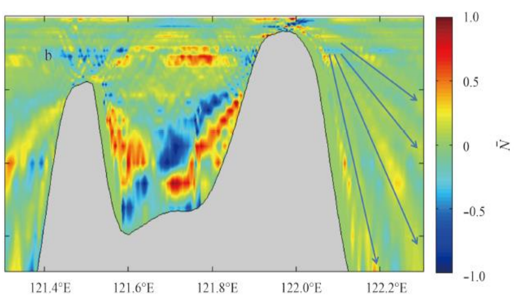
Аттрактор в Лусонском проливе



Лучевой скелет аттрактора
в Лусонском проливе
Tang & Peacock (Chaos 2010)

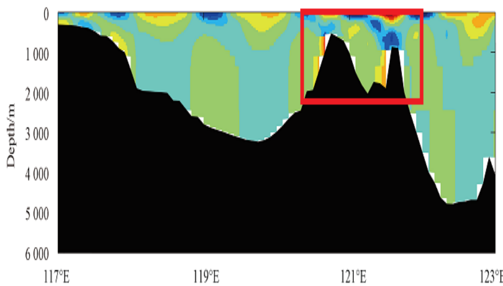


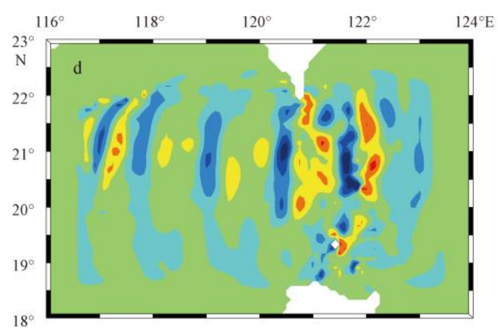
Распределение частоты
плавучести в разные сезоны



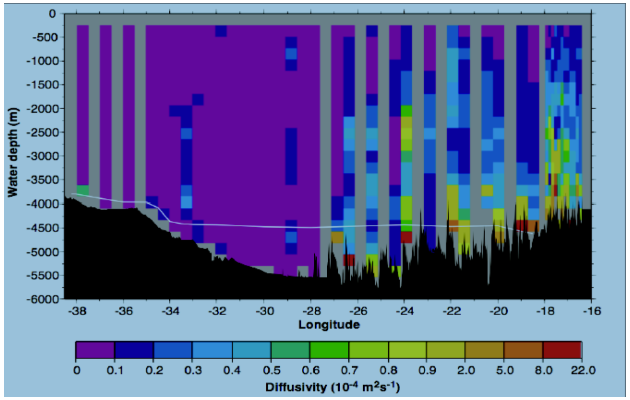
Поле возмущений частоты
плавучести

Wang G., Zheng Q., Lin M., Qiao F. Three dimensional simulation of internal wave attractors in the Luzon strait // Acta Oceanol. Sinica. 2015. V. 34, N 11. P. 14–21.



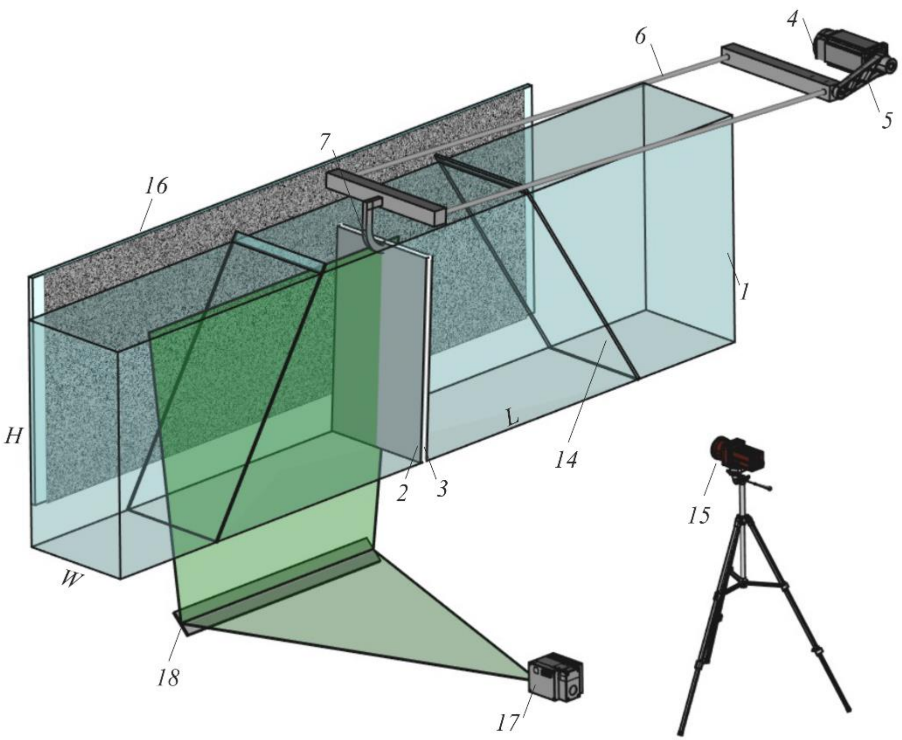


Bingtian et al.
(Acta. Oceanol. Sinica. 2015)



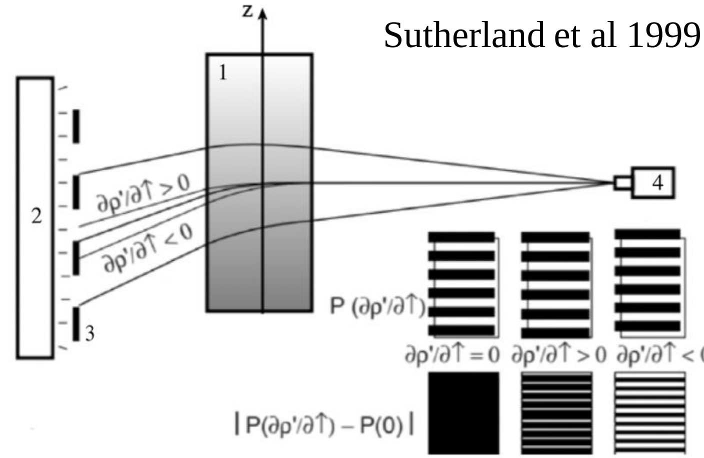
Вертикальное перемешивание
в Бразильской котловине
Polzin *et al*. (Science 1997)

Квазидвумерная постановка



1 – лоток; 2 – рама волнопродуктора; 3 – пластина волнопродуктора; 4 – шаговый двигатель; 5 – кривошип; 6 – шатун; 7 – поводок; 14 – рефлектор; 15 – камера, 16 – экран шлирена, 17 – лазер, 18 – зеркало.

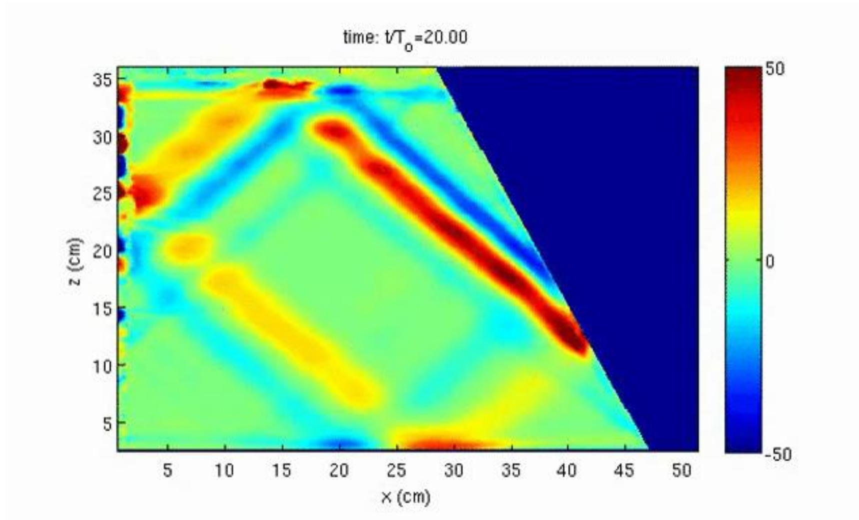
Sutherland et al 1999



Screen distance

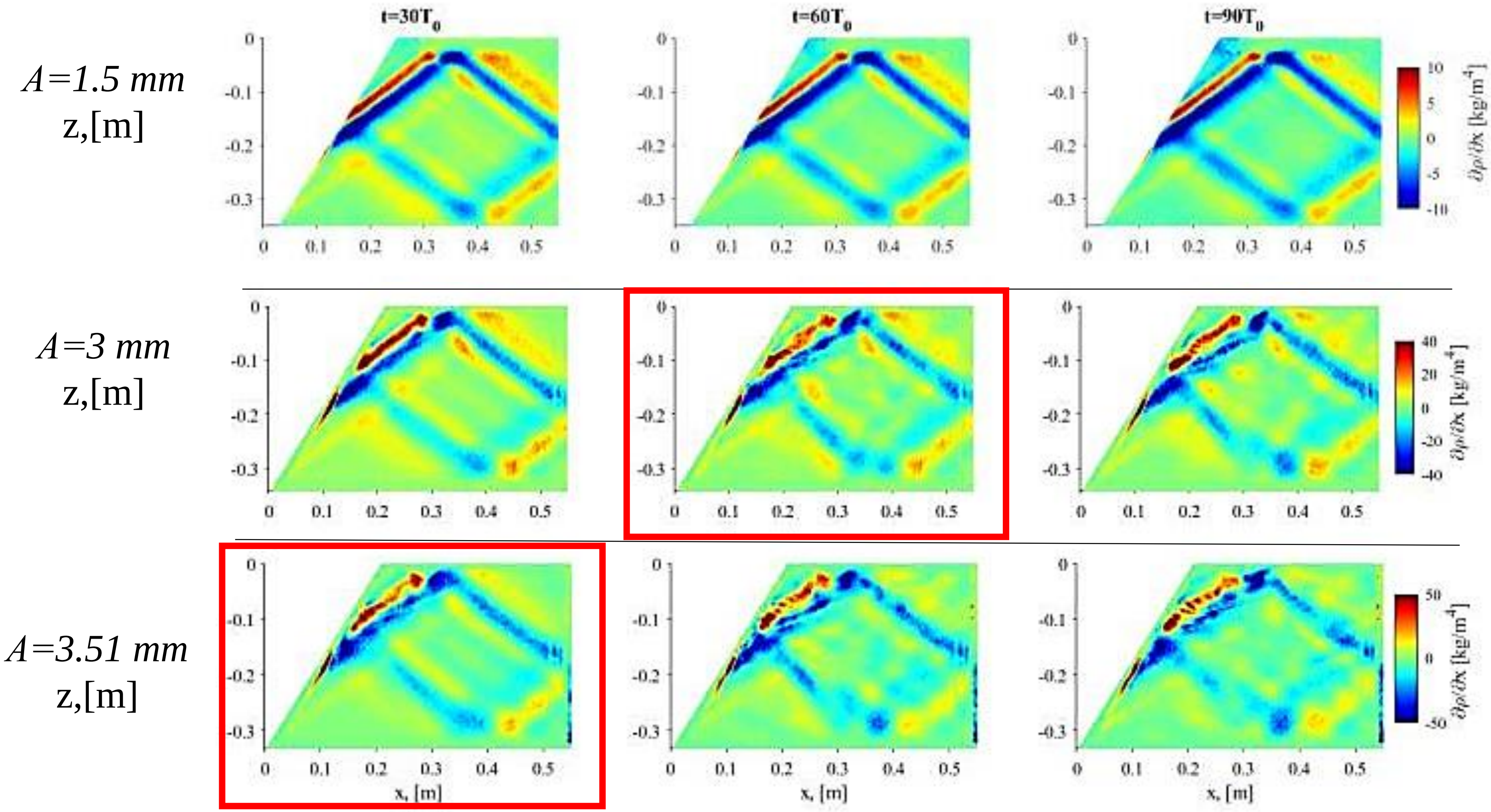
Camera distance

$$\frac{\partial\rho}{\partial z} = r\Delta p \frac{d\rho}{dn}\frac{n}{W}\frac{B+W+D}{\left(B+\frac{W}{2}\right)}$$



(1,1) attractor

Instability development



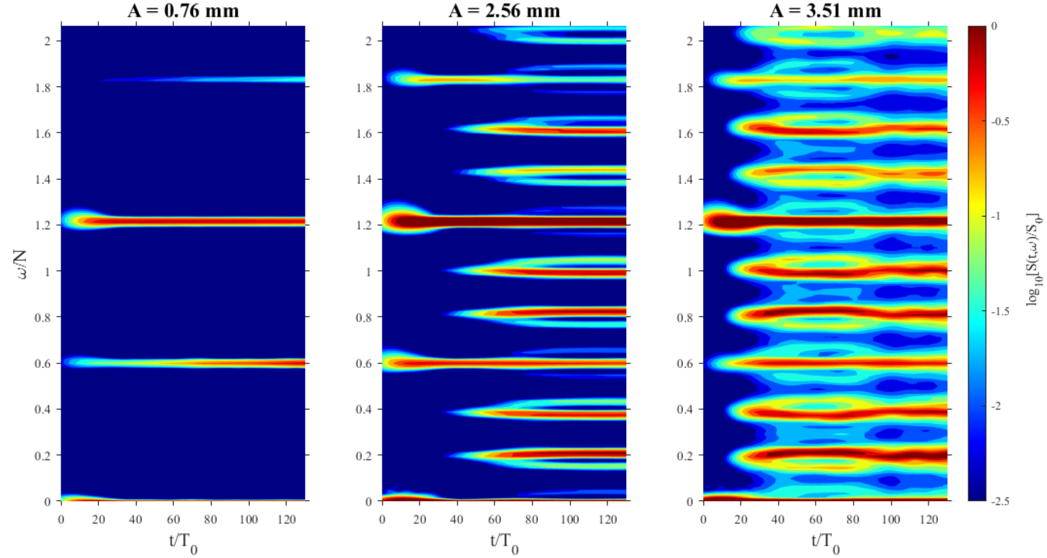
➢ Higher the amplitude – earlier the nonlinearity

Time – frequency spectrum

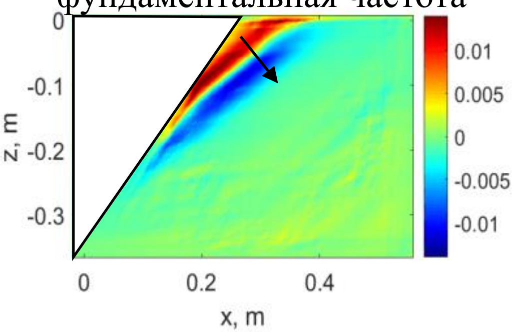
$$S(\omega, t) = \left\langle \int_{-\infty}^{\infty} \rho(\tau) e^{-i\omega\tau} h(t-\tau) d\tau \right\rangle$$

P. Flandrin, 1999

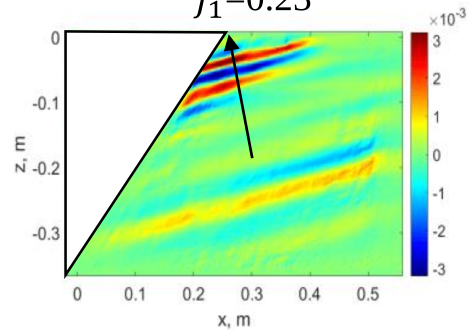
$\rho(t)$ - density disturbance



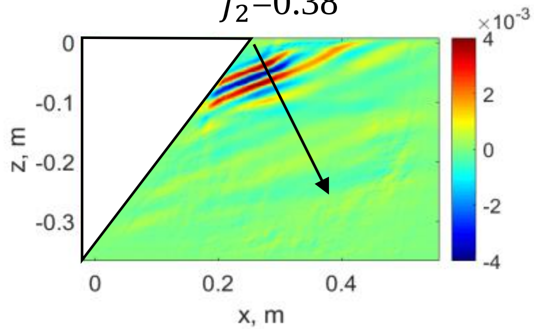
$f_0 = \omega/N = 0.63$
фундаментальная частота



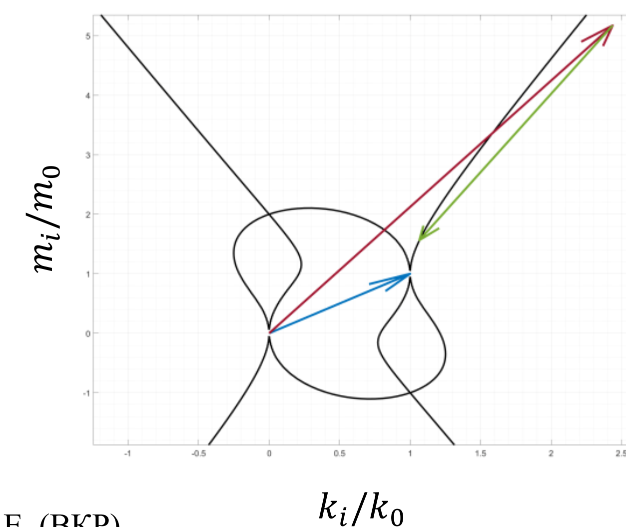
f_1=0.23



f_2=0.38



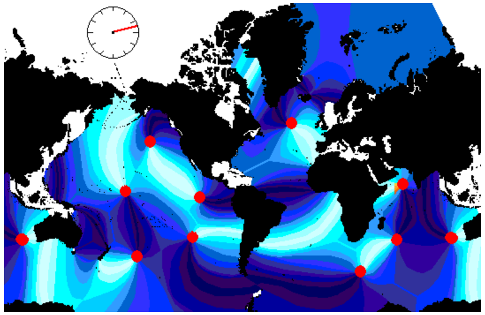
A=2.56 mm



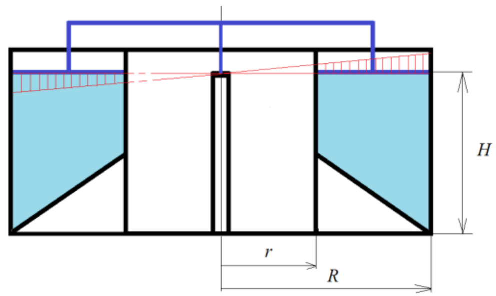
Замараева Е. (ВКР)

- Wave turbulence development due to TRI

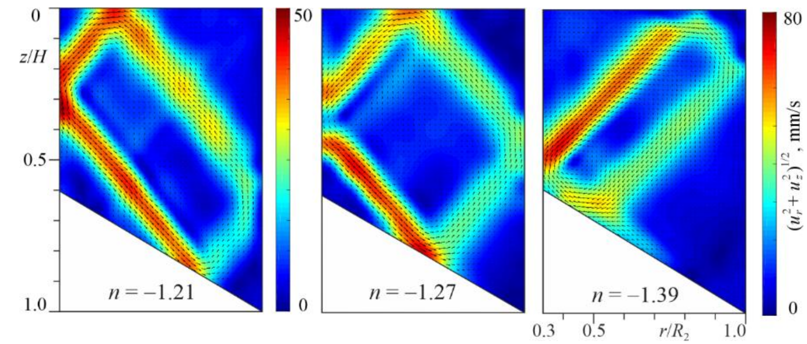
Inertia waves



Возмущение с помощью прецессии крышки
эксперимент в Перми



Вид атрактора в линейном режиме при разных частотах.



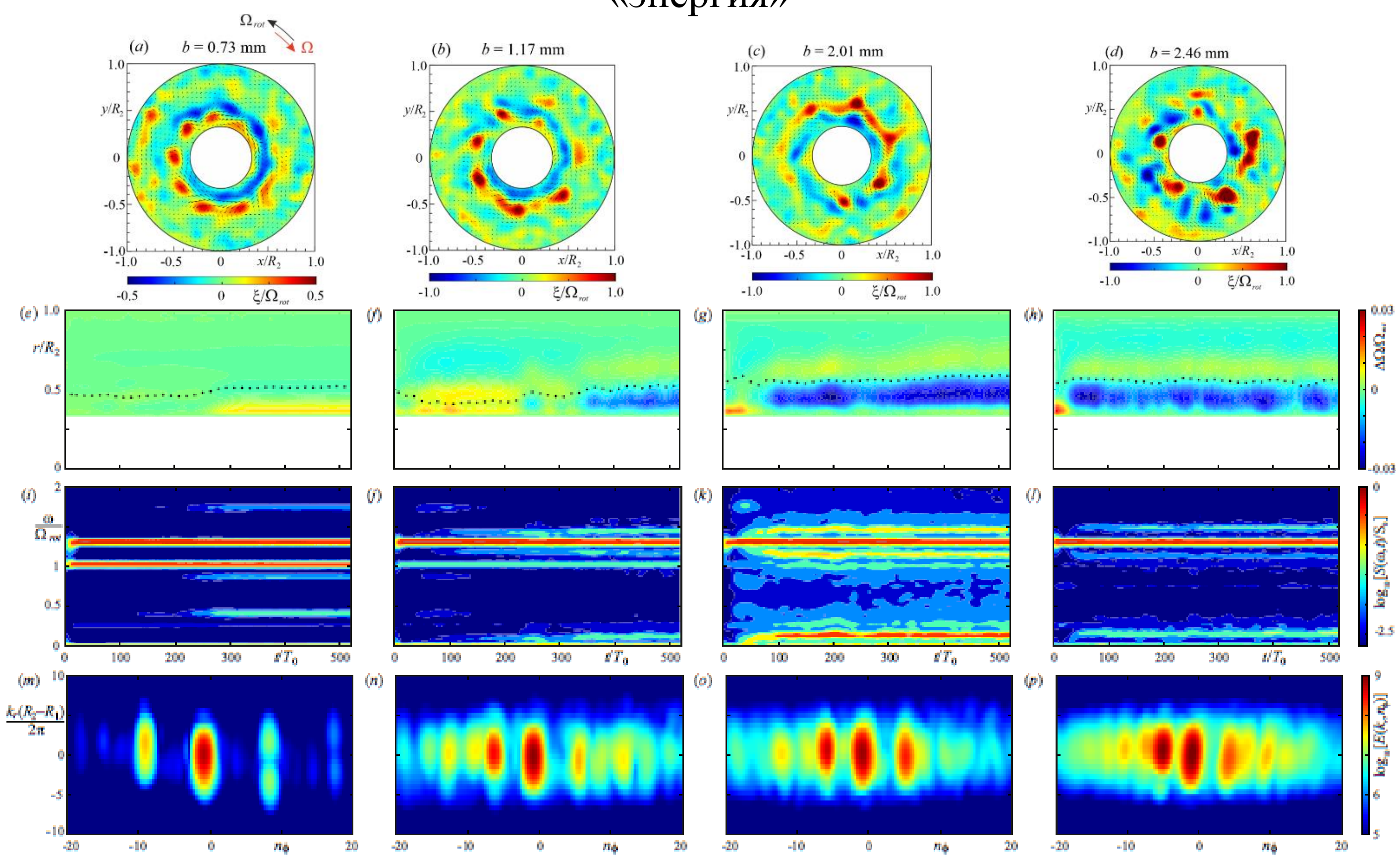
«энергия»

Картина течения

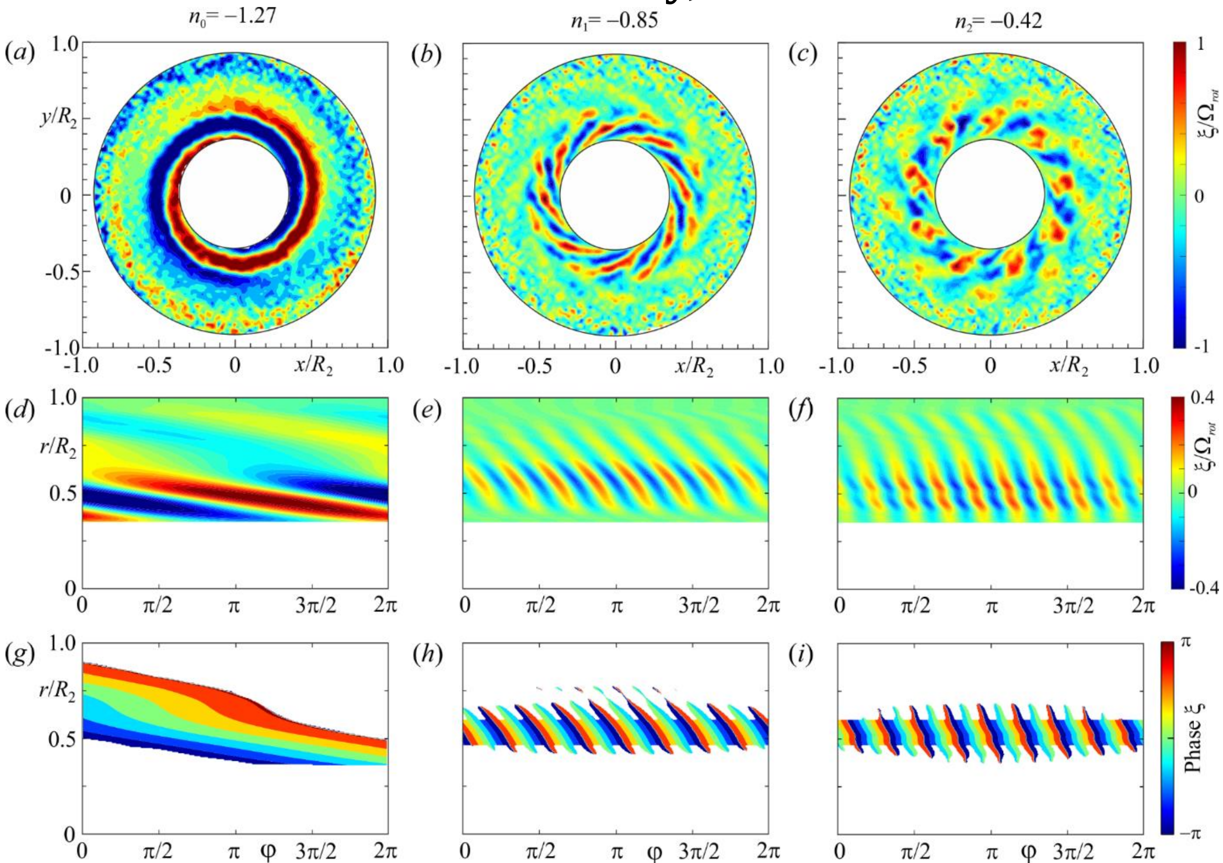
Среднее азимутальное течение

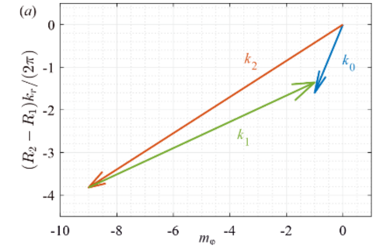
Частотный спектр

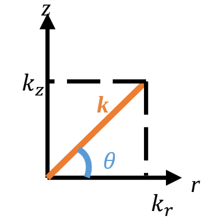
Пространственный спектр в развитом режиме



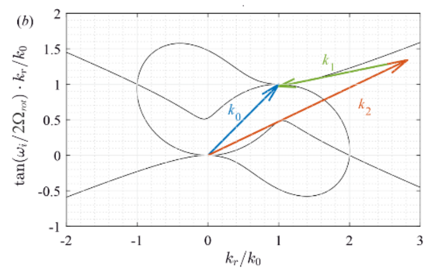
Triadic resonance instability, $b = 0.73\ mm$







$$\frac{\omega}{2\Omega_{rot}} = sin\theta \ \rightarrow k_z = \tan\left(\frac{\omega_i}{2\Omega_{rot}}\right) \cdot k_r$$



Пространственный спектр

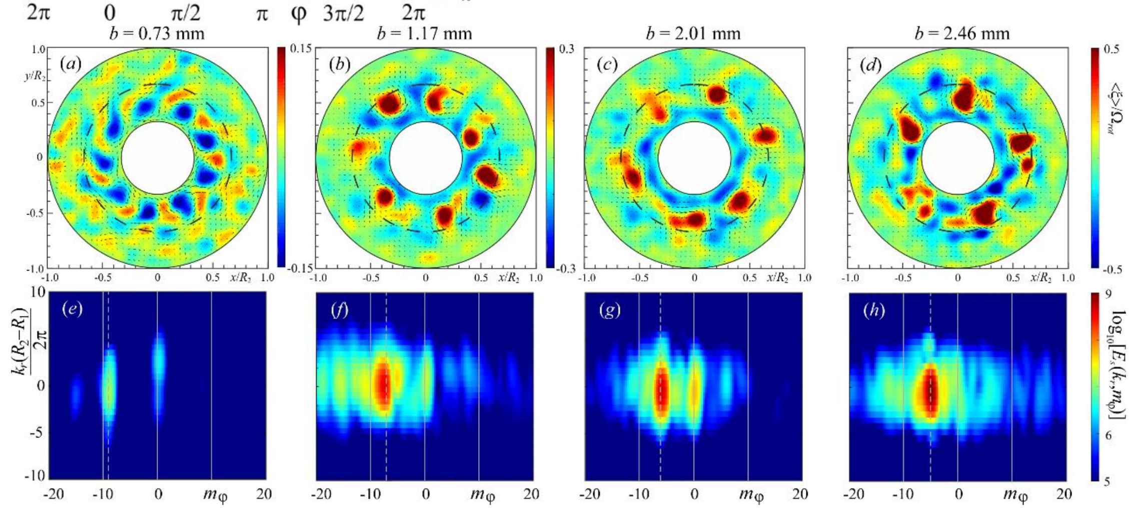
2D PIV поле скорости $v_r(r,\phi,t)$ и $v_\phi(r,\phi,t)$

3D преобразование Фурье: $\hat{v}_r\ (r,\phi,t)$ и $\hat{v}_\phi(r,\phi,t)$

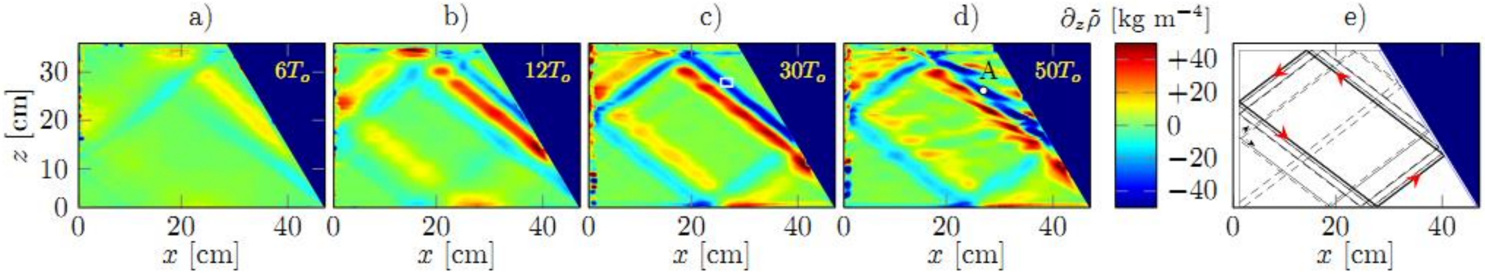
Спектр энергии:

$$E(k_\mathrm{r}, \mathrm{m}_\phi, \omega) = \frac{|\hat{v}_r(k_\mathrm{r}, m_\phi, \omega)|^2 + |\hat{v}_\phi(k_r, m_{phi}, \omega)|^2}{2ST}$$

Фильтр на низких частотах $0 > \omega > n_0/3$

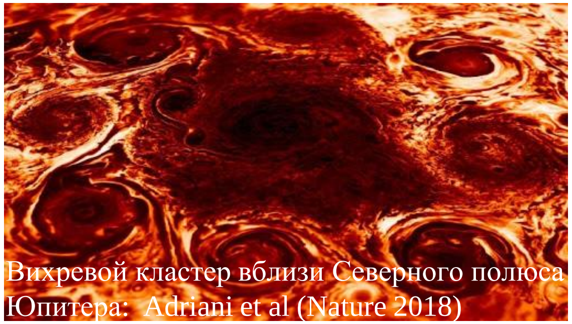


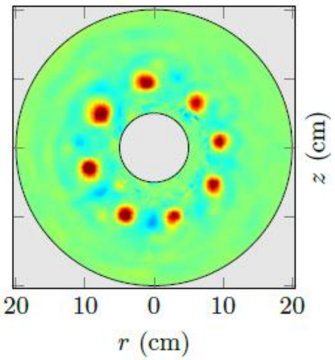
Wave turbulence to vortex clusters

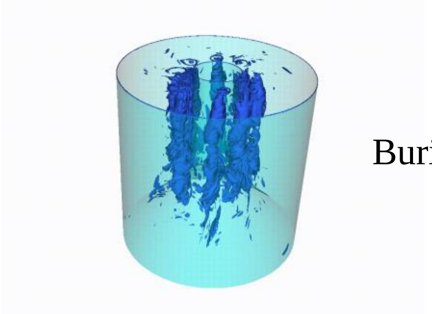


Scolan et al 2013

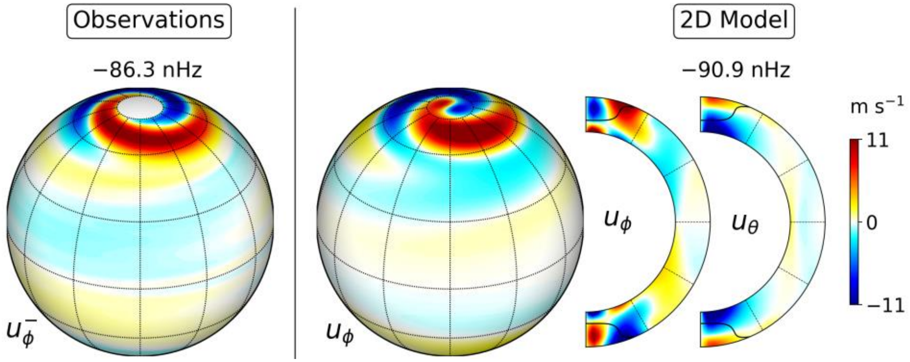
➢ Triadic resonant instability to wave turbulence – slow mixing and vortex cluster development


Вихревой кластер вблизи Северного полюса Юпитера: Adriani et al (Nature 2018)

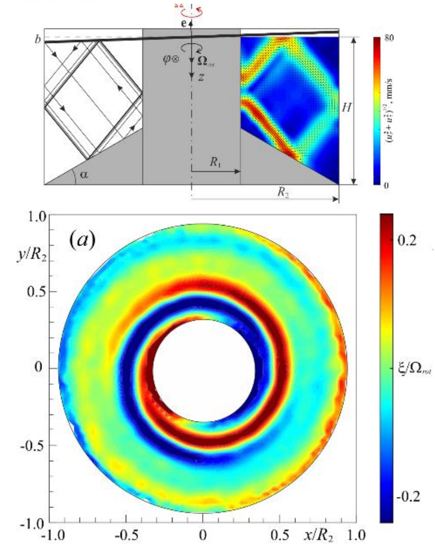




Buri et al. 2021



Недавние открытия в области гидродинамики Солнца:
Экваториальные волны Россби: Loptien et al (Nature Astronomy 2018),
Конвективный слой Солнца – одна ячейка в каждом полушарии: Gizon et al (Science 2020)
Инерционные волны в конвективном слое Солнца: Gizon et al (A&A, 2021) Bekki, Cameron, Gizon (A&A, 2022)



Subbotin et al. 2023 (rev)

Спасибо за внимание!