

LINEAR PROGRAMMING AND KANTOROVICH SPACES

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ABSTRACT. This is a brief overview of the life of Leonid Kantorovich (1912–1986) and his contribution to the fields of linear programming and ordered vector spaces.

January 19, 2007 is the date of the 95th anniversary of the birth of Leonid Kantorovich. Among the Russian scientists, Leonid Kantorovich and Vasily Leontiev were the only persons awarded with a Nobel prize in economics. Kantorovich occupies a rather special place in the history of the world science, ranking among the giants that synthesized the exact and humanitarian modes of thought in their creative activities. Kantorovich was elected a corresponding member of the Academy of Sciences of the USSR in economics and a full member in mathematics. There are no persons with similar biographic signposts in the national academy of this country. Kantorovich was among the scientists that comprised the first staff of the Siberian Division of the Academy of Sciences of the USSR and spent a decade in Akademgorodok near Novosibirsk.

1. THE WORLD LINE OF KANTOROVICH

Kantorovich was born in the family of a venereologist at St. Petersburg on January 19, 1912 (January 6, according to the old Russian style). It is curious that many reference books give another date (which is three days before). Kantorovich kept explaining with a smile that he remembers himself from January 19, 1912. The boy's talent was revealed very early. In 1926, just at the age of 14, he entered St. Petersburg (then Leningrad) State University (SPSU). Soon he started participating in a circle of G. M. Fikhtengolts for students and in a seminar on descriptive function theory. It is natural that the early academic years formed his first environment: D. K. Faddeev, I. P. Natanson, S. L. Sobolev, S. G. Mikhlin, and a few others with whom Kantorovich was friendly during all his life also participated in Fikhtengolts's circle. The old cronies called him "Lënechka" ever since these days.

After graduation from SPSU in 1930, Kantorovich started teaching, combining it with intensive scientific research. Already in 1932 he became a full professor at the Leningrad Institute of Civil Engineering and an assistant professor at SPSU. From 1934 Kantorovich was a full professor at his *alma mater*.

The main achievements in mathematics belong to the "Leningrad" period of Kantorovich's life. In the 1930s he published more papers in pure mathematics whereas his 1940s are devoted to computational mathematics in which he was soon appreciated as a leader in this country.

The letter of Academician N. N. Luzin [3], written on April 29, 1934, was found in the personal archive of Kantorovich a few years ago during preparation of his selected works for publication [1].

Key words and phrases. Linear programming, Dedekind complete vector lattice, Boolean valued analysis.

This letter demonstrates the attitude of Luzin, one of the most eminent and influential mathematicians of that time, to the brilliance of the young prodigy. Luzin was one the nest mathematicians of that epoch and the founder of the famous “Lusitania” school of Muscovites. He remarked in his letter:

... you must know my attitude to you. I do not know you as a man completely but I guess a warm and admirable personality.

However, one thing I know for certain: the range of your mental powers which, so far as I accustomed myself to guess people, open up limitless possibilities in science. I will not utter the appropriate word—what for? Talent—this would belittle you. You are entitled to get more...

From the end of the 1930s the research of Kantorovich acquires new traits as made an outstanding breakthrough in economics. His booklet *Mathematical Methods in the Organization and Planning of Production* that appeared in 1939 is a material evidence of the birth of linear programming. The economic works of Kantorovich were hardly visible at the surface of the scientific information flow in the 1940s. However, the problems of economics prevailed in his creative studies. During the Second World War he completed the first version of his book *The Best Use of Economic Resources* which led to the Nobel Prize awarded to him and Tjalling C. Koopmans in 1975.

In 1957 Kantorovich accepted the invitation to join the newly founded Siberian Division of the Academy of Sciences of the USSR. He agreed and soon became a corresponding member of the Department of Economics in the first elections to the Siberian Division. Since then his major publications were devoted to economics, with the exception of the celebrated course of functional analysis [4], “Kantorovich and Akilov” in the students’ jargon.

I cannot help but mention one brilliant twist of mind of Kantorovich and his students in suggesting a scientific approach to taxicab metered rates. The people of the elder generation remember that in the 1960s the taxicab meter rates in the USSR were modernized radically: there appeared a price for taking a taxicab which was combining with a less per kilometer cost. This led immediately to raising efficiency of taxi parks as well as profitability of short taxicab drives. This economic measure was a result of a mathematical modeling of taxi park efficiency which was accomplished by Kantorovich with a group of young mathematicians and published in the most prestigious mathematical journal “Russian Mathematical Surveys.”

The 1960s became the decade of his recognition. In 1964 he was elected a full member of the Department of Mathematics of the Academy of Sciences of the USSR, and in 1965 he was awarded the Lenin Prize. In these years he vigorously propounded and maintained his views of interplay between mathematics and economics and exerted great efforts to instill the ideas and methods of modern science into the top economic management of the Soviet Union, which was almost in vain.

At the beginning of the 1970s Kantorovich left Novosibirsk for Moscow where he was still engaged in economic analysis, not ceasing his efforts to influence the everyday economic practice and decision making in the national economy. These years witnessed a considerable mathematical Renaissance of Kantorovich. Although he never resumed proving theorems, his impact on the mathematical life of this country increased sharply due to the sweeping changes in the Moscow academic life on the eve of Gorbi’s “perestroika.” Cancer terminated his path in science on April 7, 1986. He was buried at the Novodevichy Cemetery in Moscow.

2. CONTRIBUTION TO SCIENCE

The scientific legacy of Kantorovich is immense [1]. His research in the areas of functional analysis, computational mathematics, optimization, and descriptive set theory has had a dramatic impact on the foundation and progress of these disciplines. Kantorovich deserved his status of one of the father founders of the modern economic-mathematical methods. Linear programming, his most popular and celebrated discovery, has changed the image of economics.

Kantorovich wrote more than 300 articles. When we discussed with him the first edition of an annotated bibliography of his publications in the early 1980s, he suggested to combine them in the nine sections: descriptive function theory and set theory, constructive function theory, approximate methods of analysis, functional analysis, functional analysis and applied mathematics, linear programming, hardware and software, optimal planning and optimal prices, and the economic problems of a planned economy.

The impressive diversity of these areas of research rests upon not only the traits of Kantorovich but also his methodological views. He always emphasized the innate integrity of his scientific research as well as mutual penetration and synthesis of the methods and techniques he used in solving the most diverse theoretic and applied problems of mathematics and economics. I leave a thorough analysis of the methodology of Kantorovich's contribution a challenge to professional scientometricians. It deserves mentioning right away only that the abstract ideas of Kantorovich in the theory of Dedekind complete vector lattices, now called *Kantorovich spaces* or *K-spaces*,¹ turn out to be closely connected with the art of linear programming and the approximate methods of analysis.

It is impossible to shed light on everything invented by Kantorovich in a short article. He told me once that his main mathematical achievement is the development of the theory of *K-spaces* within functional analysis while remarking that his most useful deed is linear programming. These beautiful pearls of the scientific legacy of Kantorovich deserve a special overview.

3. LINEAR PROGRAMMING

Linear programming belongs to the curricula of hundreds of thousands of students throughout the world. This term signifies a colossal scientific area that addresses linear models of optimization. In other words, the science and art of linear programming deal with a theoretical and numerical analysis as well as solution of the problems in which we seek for an optimal value, i. e. a maximum or minimum, of some system of indicators in any process whose states and/or behavior are determined by simultaneous linear inequalities. The term "linear programming" was suggested in 1951 by Koopmans who contributed greatly to proliferation of the idea of linear programming and defending the priority of Kantorovich in this area.

In the USA, linear programming started only in 1947 by George B. Dantzig who convincingly described the history of linear programming in his classical book [2, p. 22–23] as follows:

*The Russian mathematician L. V. Kantorovich has for a number of years been interested in the application of mathematics to programming problems. He published an extensive monograph in 1939 entitled *Mathematical Methods in the Organization and Planning of Production*...*

¹Kantorovich wrote about "my spaces" in his personal memos.

Kantorovich should be credited with being the first to recognize that certain important broad classes of production problems had well-defined mathematical structures which, he believed, were amenable to practical numerical evaluation and could be numerically solved.

In the first part of his work Kantorovich is concerned with what we now call the weighted two-index distribution problems. These were generalized first to include a single linear side condition, then a class of problems with processes having several simultaneous outputs (mathematically the latter is equivalent to a general linear program). He outlined a solution approach based on having on hand an initial feasible solution to the dual. (For the particular problems studied, the latter did not present any difficulty.) Although the dual variables were not called “prices,” the general idea is that the assigned values of these “resolving multipliers” for resources in short supply can be increased to a point where it pays to shift to resources that are in surplus. Kantorovich showed on simple examples how to make the shifts to surplus resources. In general, however, how to shift turns out to be a linear program in itself for which no computational method was given. The report contains an outstanding collection of potential applications...

If Kantorovich’s earlier efforts had been appreciated at the time they were first presented, it is possible that linear programming would be more advanced today. However, his early work in this field remained unknown both in the Soviet Union and elsewhere for nearly two decades while linear programming became a highly developed art.

It is worth observing that to an optimal plan of every linear program there corresponds some optimal prices or “objectively determined estimators.” Kantorovich invented this bulky by tactical reasons in order to enhance the “criticism endurance” of the concept. The interdependence of optimal solutions and optimal prices is the crux of the economic discovery of Kantorovich.

4. KANTOROVICH SPACES

In the mid-1930s the research of Kantorovich contributed to the foundation of a new fruitful area of functional analysis, the theory of ordered vector spaces. Kantorovich introduced and elaborated the class of the so-called Dedekind complete vector lattices in which each bounded subset has a least upper bound. These spaces are often referred to as K -spaces or Kantorovich spaces. Kantorovich gave versatile applications of his theory to many problems of functional analysis, function theory, and the theory of operator equations.

Kantorovich incessantly emphasized the immanent connection between the theory of K -spaces, the theory of inequalities, and economic topics. The further research of various authors demonstrates that the idea of linear programming are inseparable from the theory of K -spaces in the following rigorous sense: The validity of each of the universally accepted formulations of the duality principle with prices in some algebraic structure necessitates that this structure is a K -space.

The development of Boolean valued model of set theory began in the 1960s in connection with the Paul Cohen’s final solution of the continuum problem. This problem was raised by David Hilbert as the firsts in the list of his epochal report to the International Congress of Mathematicians in 1900. The progress of the recent Boolean valued analysis [5] has demonstrated the fundamental significance of universally complete Kantorovich spaces. Unexpectedly, each of these spaces turns out to be one of the legitimate models of the real line and so plays the same

key role in mathematics. It is curious that the development of new logical tools led in the 1980s to reinvention of K -spaces in the USA under the title “Boolean linear spaces.”

Magically prophetic happens to be the claim of Kantorovich that the elements of a K -space are generalized numbers. The heuristic transfer principle of Kantorovich found a brilliant justification in the framework of modern mathematical logic. Guaranteeing a profusion of unbelievable models of the real axis, the spaces of Kantorovich will stay in the treasure-trove of the world science for ever.

5. SYNTHESIS OF CULTURES

Kantorovich is rightfully ranked among the founders of the economic-mathematical area. Linear programming, his best invention, changed the appearance of economics. Kantorovich was in full possession not only of the gift of a mathematical genius but also stamina of an intellectual champion in struggling for new economic theories.

The ideas and methods of linear programming gave rise to deep interdisciplinary research, trespassed the frontiers of economics, and won appreciation in the various spheres of human activities. It is difficult to distinguish another scholar in the history of the twentieth century science who contributed as much as him to interpenetration of mathematics and economics, the science with the antipodal standards of scientific thought. Israel Gelfand pointed out that he can list only John von Neumann and Andrei Kolmogorov alongside Leonid Kantorovich among his contemporaries synthesizing the mathematical and humanitarian cultures.

Alfred Marshall (1842–1924), the founder of the Cambridge school of neoclassicals, “Marshallians,” wrote in his magnum opus [6]:

The function then of analysis and deduction in economics is not to forge a few long chains of reasoning, but to forge rightly many short chains and single connecting links... [6, Appendix C: The Scope and Method of Economics. § 3.]

It is obvious that there is no room in economics for long trains of deductive reasoning. [6, Appendix D: Use of Abstract Reasoning in Economics]

In 1906 he formulated his scepticism in regard to mathematics as follows [7, p. 294]:

[I had] a growing feeling in the later years of my work at the subject that a good mathematical theorem dealing with economic hypotheses was very unlikely to be good economics: and I went more and more on the rules —

- (1) Use mathematics as a shorthand language, rather than an engine of inquiry.*
- (2) Keep to them till you have done.*
- (3) Translate into English.*
- (4) Then illustrate by examples that are important in real life.*
- (5) Burn the mathematics.*
- (6) If you can't succeed in (4), burn (3). This last I did often.*

Marshall intentionally counterpose the economic and mathematical ways of thinking, noting that the numerous short “combs” are appropriate in a concrete economic analysis. Clearly, the image of a “comb” has nothing in common with the upside-down pyramid, the cumulative hierarchy of the von Neumann universe, the residence of the modern Zermelo–Fraenkel set theory. It is from the times of Hellas that the beauty and power of mathematics rest on the axiomatic method which presumes the derivation of new facts by however lengthy chains of formal implications.

The conspicuous discrepancy between economists and mathematicians in mentality has hindered their mutual understanding and cooperation. Many partitions, invisible but ubiquitous, were erected in ratiocination, isolating the economic community from its mathematical counterpart and vice versa.

This status quo with deep roots in history was always a challenge to Kantorovich, contradicting his views of interaction between mathematics and economics. His path in science is well marked with the signposts conveying the slogan: “Mathematicians and Economists of the World, Unite!” His message has been received as witnessed by the curricula and syllabi of every economics department in a major university throughout the world.

Despite the antediluvian opinion that “the mathematical scientist emperor of mainstream economics is without any clothes” (cp. [8]), the gadgets of mathematics and the idea of optimality will come in handy for the working economist. Calculation will supersede prophecy. Economics as a boon companion of mathematics will avoid merging into any esoteric part of the humanities, or politics, or belles-lettres. The new generations of mathematicians will treat the puzzling problems of economics as an inexhaustible source of inspiration and an attractive arena for applying and refining their formal methods.

The life of Kantorovich is a path of a scholar and citizen whose scientific work is inseparable from the fates of his fellows, exemplifying the idea of serving the genuine ends of one’s Fatherland regardless of whatever ideological situation. This lesson is of major import these days. Attempts at slandering or hushing up the life and legacy of Kantorovich are doomed to demolishing. Pygmies can never hide a giant...

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