ISSN 1990-4789, Journal of Applied and Industrial Mathematics, 2010, Vol. 4, No. 2, pp. 191–193. © Pleiades Publishing, Ltd., 2010.
Original Russian Text © A.E. Gutman, S.S. Kutateladze, Yu.G. Reshetnyak, 2010, published in Sibirskii Zhurnal Industrial'noi Matematiki, 2010, Vol. XIII, No. 1, pp. 55–58.

Cofinite Numbers, Nonstandard Analysis, and Mechanics

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Received December 14, 2009

Abstract—We demonstrate the mathematical insignificance of the versions of nonstandard analysis proposed in the articles by A. F. Revuzhenko.

DOI: 10.1134/S1990478910020079

Key words: *nonstandard analysis, Revuzhenko, pseudoscience*

The writings of A. F. Revuzhenko devoted to inventing new numbers [1–3] for applied problems are not the scientific base of his activity in elastoplasticity. These are independent articles which in their author's opinion introduce new elements into the technique and understanding of objects of mathematical analysis. In reality, new in these writings is an unjustifiable use of the terminology and concepts of mathematical analysis concerning the objects to which they either do not apply at all or apply not in the way the author thinks.

Revuzhenko uses the term "nonstandard analysis," sometimes cites articles in nonstandard analysis, and uses some terminology of mathematical analysis, but fails to understand the essence of constructions and foundations of nonstandard analysis. The objects he considers belong to an algebraic system that modern mathematics does not regard as the nonstandard real line since this is not what it is.

Revuzhenko considers the set of sequences of rational numbers and identifies those of them that coincide starting with some index. He refers to the objects in the resultant set as "cofinite numbers." (In the usual scheme for constructing the set of real numbers going back to G. Cantor, the Cauchy sequences of rational numbers are identified for which the distance between the terms of the same index tends to zero.)

The construction used by Revuzhenko is known in algebra for a long time in a more general context of filtered products. The filter considered by Revuzhenko is called the Fréchet filter in mathematics. The analog of this construction for an arbitrary infinite set is usually called the "cofinite filter" on the set. Therefore, Revuzhenko repeats a standard mathematical construction. The use of the specific term "cofinite" is difficult to dismiss as a random coincidence; therefore, the author is aware that his construction is known in mathematics for a long time.

The branch of modern mathematics called "nonstandard analysis" uses an ultraproduct with an ultrafilter including the Fréchet filter. It is well known that every filter can be obtained as an intersection of ultrafilters. The use of ultrafilters is a key to the success of the constructions originally made by the outstanding American logician and gas dynamicist A. Robinson [4], who created delta-wing theory [5, 6] and nonstandard analysis [7, 8].

Consider the two sequences a and b defined as

$$a(2n-1) = \frac{1}{2n-1}, \qquad a(2n) = 0, \qquad b(2n-1) = 0, \qquad b(2n) = \frac{1}{2n},$$

i.e.,

$$a = \big(1, 0, \frac{1}{3}, 0, \frac{1}{5}, 0, \frac{1}{7}, 0, \dots\big), \qquad b = \big(0, \frac{1}{2}, 0, \frac{1}{4}, 0, \frac{1}{6}, 0, \dots\big),$$

and converging to zero. According to Revuzhenko, these sequences cease to be equivalent, which they are according to Cantor. They determine different nonzero "cofinite numbers" in Revuzhenko's

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definition. However, their product ab is equal to zero, while neither factor is. In mathematics the objects of this type are called nontrivial zero divisors. Numbers in mathematics constitute a field, and fields by definition contain no nontrivial zero divisors. According to Robinson, these sequences are also inequivalent, but one of them is equal to zero, and the other is not. The reason is that every ultrafilter on the set of nonnegative integers contains either the set of all even numbers or the set of all odd numbers.

The system of real numbers, whose properties are enriched by nonstandard analysis, is not just an algebra, but it enjoys a series of additional properties. Revuzhenko's system of "cofinite numbers" lacks these properties, in contrast to the nonstandard real line of Robinson. One of the most important results of Robinson's nonstandard analysis is the transfer principle. According to this principle, every statement of usual mathematics valid for all standard elements of an infinite set holds for its nonstandard elements as well. This is the principle that makes nonstandard analysis particularly appealing. The constructions of Revuzhenko include no inkling of the transfer principle. Moreover, in his "analysis" this principle is utterly impossible. The successes in the application of nonstandard analysis of Robinson have to do precisely with his uses of very fine methods and reasoning of model theory, which Revuzhenko considers completely superfluous. This includes the concept of ultrafilter, lying at the foundation of the theory of Robinson but completely unnecessary to Revuzhenko. Therefore, Revuzhenko specifies no new classes of objects and brings nothing meaningful into mathematics. (Moreover, the list of various pre-Robinson versions of nonstandard analysis already includes a model based precisely on the factorization of numerical sequences by the cofinite filter [10, 11].)

These circumstances were pointed out to Revuzhenko many years ago in a negative review of his writings, one of which he had sent to Siberian Mathematical Journal. The article was rejected, but Revuzhenko published his texts in other publications of the Siberian Division of the Russian Academy of Sciences, included them in the book [1] pretentiously entitled "Mechanics of Elastoplastic Media and Nonstandard Analysis", and keeps breeding mathematical trivialities in mechanics and technology oriented publications, in particular, in the journal "Physical Mesomechanics" [2, 3].

There is no need to trace the evolution of Revuzhenko's views, in which he paves his way from studying "functions with structure" through his own "nonstandard analysis" to "cofinite," "extraordinary," and other "numbers," and the concept of a "non-Archimedean multiscale space." However, although his latest articles include attempts to examine zero divisors and similar obstacles, still there is no understanding of the difference between new models and not-quite-new generalizations and analogs of the number field, as myriads of various algebraic systems. The lack of understanding of the basis of model theory fails to check the level of ambitions based on the proposition that "the concept of real line determines the main properties of space and time" (see [3], p. 46); therefore, "considering non-Archimedean lines as the coordinate axes, we arrive at non-Archimedean space and time" (see [3], p. 54).

No incantations can carry "functions with structure" beyond the limits of trivial examples of function algebras. No references to mechanics of elastoplastic media can justify the unethical use of the term "nonstandard analysis" which is established in science in regard to trivialities; to neither of which it is possible to apply the word "nonstandard" nor the word "analysis." Both the cofinite numbers and their modifications and generalizations remain uninteresting objects of superficial discourse.

Revuzhenko's improper use of the term "nonstandard analysis" is difficult to justify. We should stress once more that there is no nonstandard analysis in the writings of Revuzhenko, but there are speculations on related topics. The reason is that the problem solved by Robinson in his nonstandard analysis consisted not in how to enrich the number system by invoking new elements, but in how to introduce actually infinitely large and small elements into classical calculus without losing the essential properties of the usual real numbers. Robinson's nonstandard analysis ranks among the most outstanding achievements of mathematics of the twentieth century. Already in his classical book Robinson gave applications of nonstandard analysis to the derivation of boundary layer equations and the Saint Venant principle (see [8], 9.6, 9.7). During the following half a century the ideas of Robinson demonstrated their effectiveness in a series of applications (for instance, see [12–17]).

The writings of Revuzhenko include no serious applications of his "cofinite numbers." This is not surprising since the technique of Revuzhenko is mathematically void and thus actually fruitless. Thus, the elasticity equations which Revuzhenko obtains using his approach precisely coincide with those familiar in mechanics. This is also predictable: the classics who established the main equations of continuum mechanics, beginning with Euler and up to our time, in their arguments widely used infinitely small quantities in the form expressed by Newton and Leibniz.

Without invoking the concept of ultrafilter or its equivalents, the required extension of the number field is impossible to achieve, and this fact is well known in mathematics for a long time. The writings of Revuzhenko belong to the class of vacuous and unsuccessful attempts to develop and generalize non-standard analysis ignoring its formal techniques, and in particular, model theory techniques. At the same time, modern nonstandard set theory [18] has already had for a long time ready-to-use tools enabling us to model the "multiscale space" and "hierarchy of structural levels" and, moreover, both analytically (using infinitesimals of different orders) and logically (via the concept of relative standardness [19]). Meanwhile the depth of the writings of Revuzhenko is at most at the level of the first pages of popular expositions of the foundations of nonstandard analysis since the author has failed to understand the scientific problems arising here and master the available techniques of mathematical analysis. The flow of pseudoscientific publications by Revuzhenko with reference to nonstandard analysis has, regretfully, been unceasing for more than 10 years.

It is sad that writings combining pretentiousness and ignorance find their way into the realm of academic publication. Unfortunately, all articles by Revuzhenko concerned with inventing new numbers belong to this class.

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