# Viktor Abramovich Zalgaller
(obituary)

On 2 October, 2020, just over two months before his centenary, the outstanding representative of the St Petersburg School of Geometry, honourary member of the St Petersburg Mathematical Society, Professor Viktor Abramovich Zalgaller passed away.

He was born on 25 December 1920 in the village of Parfino, Novgorod Province, in the family of engineer Abram Leont'evich Zalgaller and attorney Tat'yana Markovna Shabad-Zalgaller. In 1922 the family moved to Petrograd. In 1931 his father was convicted under Article 58[1] and spent 16 years in Ukhtpechlag[2], in prison, and then in exile.

After graduating from School no. 103 in the Smol'ninskii district of Leningrad in 1937, Viktor enrolled in the Faculty of Mathematics and Mechanics of Leningrad University. His abilities were noticed by L. V. Kantorovich, his lecturer on mathematical analysis, who asked the third-year student to prepare a textbook based on his notes.

In 1940, as part of a mobilisation announced by the Young Communist League, Zalgaller was transferred to the Aviation Institute. There he received an engineering education, which played a significant role in his further research career.

In the first days of the war he volunteered for the People's Militia (Second Division) and was sent to the front almost immediately. During almost the entire war he was a signaller. The combat path of Viktor Zalgaller was not easy: the defence of Leningrad, the Oranienbaum Bridgehead, an injury during the lifting of the Leningrad blockade, the storming of Vyborg, battles in the Baltic states, the storming of Danzig, reaching the Elbe... Zalgaller was awarded the Order of the Red Star, the medal "For Courage", three medals "For Battle Merit", the medal "For the Defence of Leningrad", and others. He met the end of the war as

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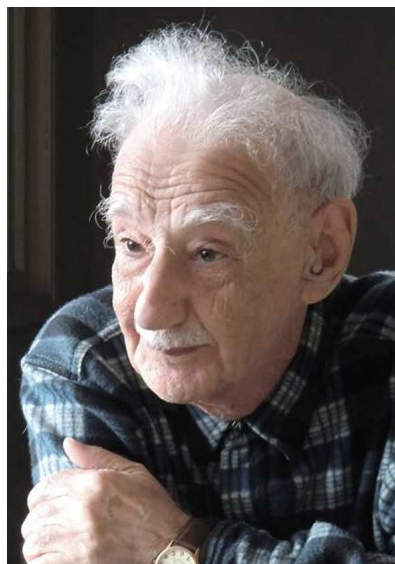
[1] 'Counter-revolutionary activity'.

[2] A subdivision of Gulag.

AMS 2020 *Mathematics Subject Classification.* Primary 01A70.

a senior sergeant. His younger brother, Lieutenant Leonid Zalgaller, died in 1942 after his unit was surrounded near Myasnoi Bor.

After demobilisation Zalgaller returned to the 3rd year courses at the Faculty of Mathematics and Mechanics. In 1948 he graduated with honours from the Department of Geometry of the Faculty. In the same year he joined the Leningrad Department of the Steklov Mathematical Institute of the USSR Academy of Sciences (LDMI, now PDMI), where he worked for more than 50 years. Beginning in 1950, he also gave lectures on a part-time basis at the Faculty of Mathematics and Mechanics of Leningrad (now St Petersburg) State University. In 1999, for family reasons he moved to Israel, were he was a consultant at the Weizmann Institute of Science in Rehovot. Conferences were organised there to celebrate his 80th and 90th birthdays.

Zalgaller understood engineering problems well and knew how to speak a common language with engineers. In the first years after graduation, he actively collaborated with Kantorovich, who engaged him in the work on optimal cutting of materials and sent him to the Leningrad Carriage Works to test his ideas in practice. Their joint activity culminated in the publication of the book *Calculation of rational cutting of industrial materials* (Lenizdat, Leningrad, 1951, 206 p.; 3rd ed.: Nevsky Dialect, St. Petersburg, 2012). That book turned out to be both mathematically interesting and extremely useful in specific problems of cutting metal, timber, leather, and so on. In it, many topics on linear programming and its applications were covered for the first time in the world literature. In particular, the approach proposed by Zalgaller anticipated some ideas in dynamic programming developed by R. Bellman.

Subsequently, engineers often turned to Zalgaller for advice. In the late 1950s, he taught calculus to engineers at refresher courses, and the audience almost doubled during this time. He always said that a person's engagement was important to him and was never discouraged by the lack of mathematical background of his interlocutor.

Now we take a look at Zalgaller's work in geometry. In 1950–1952, under the influence of his teacher A. D. Aleksandrov, he published a series of papers on the theory of curves with bounded variation of rotation and on the intrinsic geometry of convex surfaces. Very quickly he became a key participant in the famous Aleksandrov seminar.

Developing Aleksandrov's pioneering ideas, Zalgaller constructed a theory of two-dimensional manifolds of bounded curvature. In 1963 he was awarded a D.Sc. degree on the basis of a collection of his work.

An important result of Zalgaller's is related to the Nash–Kuiper theorem. It asserts that every d-dimensional polyhedron admits a piecewise linear intrinsic isometry to d-dimensional Euclidean space. He proved this for $d = 2, 3, 4$, but later his methods made it possible to generalise the theorem to an arbitrary dimension. In the 1990s Zalgaller revisited this subject and proved (together with Yu. D. Burago) the existence of piecewise linear isometric embeddings and immersions of two-dimensional polyhedral surfaces in three-dimensional space.

Zalgaller turned repeatedly to the theory of convex sets. He was interested in the connections between the properties of the intrinsic metrics of convex surfaces and their extrinsic geometry. This also included investigations of the behaviour of geodesics on a convex surface and, in particular, the construction of the first example of a geodesic that does not pass through ribbed points, but has a non-rectifiable spherical image. In addition, he developed applications of the theory of convex sets to probabilistic problems, in particular to the problem of the probability of falling into a convex region under a normal distribution. The theory of mixed volumes was instrumental there. The joint study by Zalgaller and V. N. Sudakov of an extremal problem related to the normal distribution in the case of a centrally symmetric convex body should also be noted in this connection.

Every mathematician who understands the classification of Platonic solids comes up with the question: what are the convex polyhedra all of whose faces are regular (not necessarily congruent) polygons? This question is much more complicated than it might seem at first glance, and Zalgaller was responsible for its solution: there are exactly 28 simple (not composed of others) regular-faced polyhedra, not counting prisms and antiprisms.

Many works by Zalgaller were devoted to extremal problems for convex sets. He worked on this immense topic throughout his life. We note a remarkable result, which he achieved during his stay in Israel: together with A. Yu. Solynin, he proved an old conjecture of Pólya and Szegö, stating that the logarithmic capacity of a planar n-gon of prescribed area attains a minimum at a regular n-gon. Their proof is beautiful and not easy. It consists of two parts, geometric and analytic, and the first part was due to Zalgaller. A very special triangulation of the n-gon was required here. As Zalgaller told his colleagues, he pondered about this problem continuously for more than a year, and in the end its solution was revealed to him in a dream. The proof of this conjecture was one of the best gifts for his 80th birthday. In 2004 this result was published in the prestigious *Annals of Mathematics*.

Zalgaller's last mathematical publication appeared in 2012.

He was the author of several books in the 'gold reserve' of mathematics literature. The most famous ones, the favorites of geometers, are the following (in addition to the already mentioned monograph on material cutting):

• A. D. Aleksandrov and V. A. Zalgaller, *Two-dimensional manifolds of bounded curvature*: *Foundations of the intrinsic geometry of surfaces* (*Trudy Mat. Inst. Steklova,* **63** (1962), 3–262[3]). This is a fundamental study of intrinsic metrics on two-dimensional manifolds and their curvature. The book introduces one of the most important concepts of modern geometric analysis, a metric whose curvature is bounded in the integral sense.

• Yu. D. Burago and V. A. Zalgaller, *Geometric inequalities* (Nauka, Leningrad, 1980, 288 pp.; a revised English edition was published by Springer[4]). This is

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[3]English translation: *Intrinsic Geometry of Surfaces*. Translations of Mathematical Monographs, Vol. 15, American Mathematical Society, Providence, RI 1967 vi+327 pp.

[4]Grundlehren der mathematischen Wissenschaften, 285. Springer Series in Soviet Mathematics. Springer-Verlag, Berlin, 1988. xiv+331 pp.

a comprehensive encyclopaedia on various inequalities for subsets of Euclidean and Riemannian spaces, a handbook for lecturers in geometry at mathematics departments, which became the successor to the classic monograph by T. Bonnesen and W. Fenchel.

• Yu. D. Burago and V. A. Zalgaller, *An introduction to Riemannian geometry* (Nauka, St Petersburg, 1994, 318 pp.; 2nd edition, 2018). This is the only textbook in Russian on Riemannian geometry where the presentation is from the standpoint of 'Riemannian geometry in the large', but at the same time is suitable for an initial familiarization with the subject.

Among the various problems attracting Zalgaller's attention, there was a place for 'entertaining mathematics'. For example, in an article he wrote with his wife S. I. Zalgaller for the *Kvant* magazine in 1980, they presented a description of the group of transformations of Rubik's Cube and proposed an algorithm for solving the cube.

Zalgaller was an outstanding teacher with an extraordinary ability to visualise the most complex ideas. He was particularly distinguished by his friendliness towards students and young people in general. Several generations of students remember him as the best lecturer of their courses. His most famous immediate student is G. Ya. Perelman.

We cannot escape mentioning V. A. Zalgaller's social activities for the benefit of mathematics and mathematical education, activities that flowed organically from his nature. In the early 1950s he was an active participant in the struggle against the attempts of Marxist philosophers to declare entire areas of mathematics 'pseudoscience'. He was one of the authors and editors of the outstanding book *Mathematics, its content, methods, and significance* (1953). Almost forty years later he played an important role in protecting the LDMI building from seizure attempts.

Zalgaller had always loved working with schoolchildren and students. Back in 1946, he started working in the Palace of Pioneers and Schoolchildren as the head of a mathematical study group. "I had good boys", he recalled, "Yura Reshetnyak, Garal'd Natanson, Misha Solomyak – all of them are boys from my study group". (Subsequently, all three became prominent mathematicians.) He was among the organisers of the specialised mathematical School no. 239, one of the first in the Soviet Union, for which he developed the first curriculum and where he taught in 1961–1962. Subsequently, he maintained close contact with this school. In his monograph *Convex Polyhedra with Regular Facets* (*Zap. Nauchn. Sem. Leningrad. Otdel. Mat. Inst. Steklov.* **2** (1967), 5–221), which he dedicated to his teacher Aleksandrov, the author thanked 12 students of School no. 239 who had helped him in his calculations. This is a rare case when schoolchildren are thanked in a serious piece of research work.

Zalgaller's literary talents deserve separate mention. His memoirs *The everyday life of war*, written in wonderful unsophisticated language, became widely known. He wrote excellent essays about his senior colleagues and teachers, I. P. Natanson, Kantorovich, and Aleksandrov. In the 1970s he conducted extensive archival and

literary research, which resulted in a unique work dedicated to the history of the LDMI building, which saw Pushkin, Zhukovskii, Karamzin, and the Decembrists.[5]

Colleagues, numerous students, friends, and relatives cherish the memory of Viktor Abramovich Zalgaller, a brilliant scholar of dramatic fate, an outstanding teacher, a person of exceptional morality and integrity, and at the same time just a kind and charming man.

D. Yu. Burago, Yu. D. Burago, A. L. Verner, A. M. Vershik,
M. L. Gromov, I. A. Ibragimov, S. V. Ivanov, S. V. Kislyakov,
S. S. Kutateladze, A. A. Lodkin, Yu. V. Matiyasevich,
N. E. Mnev, A. I. Nazarov, G. Yu. Panina, Yu. G. Reshetnyak,
V. A. Ryzhik, N. N. Uraltseva, and Ya. M. Eliashberg

Translated by T. PANOV

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[5]These works, as well as essays about the life and work of Zalgaller, can be found in the book *War and peace of a Leningrad mathematician. On the* 100*th anniversary of the birth of Viktor Zalgaller* (Avrora, St Petersburg, 2021).