On Computation Methods for Set-Valued solutions to Problems of Dynamics and Control

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The crucial elements of many solutions to problems of dynamics and control consist in describing related invariant sets and their dynamics. In practice this amounts to computation of forward and backward reachability tubes that turn out to be described by nonsmooth functions with values in nonconvex sets or in convex sets with non-smooth boundaries. The related theory may be based on modifications of Hamiltonian techniques to non-differentiable solutions while the computation relies on approximation of set-valued solution tubes through intersections and unions of parametrized arrays of ellipsoidal or polyhedral -valued tubes. This leads to a natural application of parallel computations. Indicated are classes of problems where the described approach appears effective.

Nonstandard Tools for Nonsmooth Analysis

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This is an overview of a few possibilities that are open by model theory in applied mathematics. Most attention is paid to the present state and frontiers of the Leonid Kantorovich ideas in the Cauchy method of majorants, approximation of operator equations with finite-dimensional analogs, and the Lagrange multiplier principle in multiobjective decision making.

— Agenda

— The Art of Calculus

— Pure and Applied Mathematics

— Challenges of the 20th Century

— Enter New Mentality

— Enigmas of Economics

— Enter the Reals

— Scalarization

— Order Omnipresent

— Enter Fermat

— Enter Hahn–Banach

— Enter Kantorovich

— Kantorovich's Heuristics

— Canonical Operator

— Support Hull

— Hahn–Banach in Disguise

— Enter Boole

— Enter Descent

— The Reals in Disguise

— Norming Sequences

— Domination

— Enter Abstract Norm

— Exeunt Abstract Norm

— Approximation

— Enter Epsilon

— Pareto Optimality

— Approximate Efficiency

— Subdifferential Halo

— Exeunt Epsilon

— Discretization

— Hypoapproximation

— Hyperapproximation

— The Hull of a Space

— The Hull of an Operator

— One Puzzling Definition

— Enter Epsilon and Monad

— Exeunt Epsilon

— State of the Art

— Vistas of the Future

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Some Approaches for Construction Exact Auxiliary Functions in Optimization Problems

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Two approaches to represent a constrained optimization problem as an unconstrained one are proposed. They can be treated in a framework of general notion of exact auxiliary functions proposed in [1]. The first approach is connected with exact penalty functions. The second one is considered for the case when a criterion function is not defined outside a feasible set. It is based on the idea of conical approximation.

A convex programming problem is considered: to find

$$f^* = \min\{f(x) : \quad x \in C\}, \tag{1}$$

where $C = \{x \in R^n : \quad h(x) \leqslant 0\}, \quad f, h : R^n \to R \cup \{+\infty\}$ are convex functions. The set $C \subseteq \mathrm{dom} f$ is supposed to be a convex compact, a feasible point $x^0 \in \mathrm{int} C$ is given, so $h(x^0) < 0$.

1. Let f, h be finite for any x. Denote $x^+ = \max\{0, \ x\}$. Consider a penalty function of the form

$$S(x, s) = f(x) + s \cdot h^+(x), \quad s \in R, \quad s \geqslant 0, \tag{2}$$