

ABSTRACT. This is a short obituary of Saunders Mac Lane (1909–2005).

**Saunders Mac Lane,
the Knight of Mathematics**

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San Francisco and April 14, 2005 form the terminal place and date of the marvellous almost centennial life of the prominent American mathematician Saunders Mac Lane who shared with Samuel Eilenberg (1913–1998) the honor of creation of category theory which ranks among the most brilliant, controversial, ambitious, and heroic mathematical achievements of the 20th century.

Category theory, alongside set theory, serves as a universal language of modern mathematics. Categories, functors, and natural transformations are widely used in all areas of mathematics, allowing us to look uniformly and consistently on various constructions and formulate the general properties of diverse structures. The impact of category theory is irreducible to the narrow frameworks of its great expressive conveniences. This theory has drastically changed our general outlook on the foundations of mathematics and widened the room of free thinking in mathematics.

Set theory, a great and ingenious creation of Georg Cantor, occupies in the common opinion of the 20th century the place of the sole solid base of modern mathematics. Mathematics becomes sinking into a section of the Cantorian set theory. Most active mathematicians, teachers, and philosophers view as obvious and undisputable the thesis that mathematics cannot be grounded on anything but set theory. The set-theoretic stance transforms paradoxically into an ironclad dogma, a clear-cut forbiddance of thinking (as L. Feuerbach once put it wittily). Such an indoctrinated view of the foundations of mathematics is false and contradicts conspicuously to the leitmotif, nature, and pathos of the essence of all creative contribution of G. Cantor who wrote as far back as in 1883 that “denn das *Wesen* der *Mathematik* liegt gerade in ihrer *Freiheit*.”

It is category theory that one of the most ambitious projects of the 20th century mathematics was realized within in the 1960s, the project of socializing set theory. This led to topos theory providing a profusion of categories of which classical set theory is an ordinary member. Mathematics has acquired infinitely many new degrees of freedom. All these rest on category theory originated with the article by S. Mac Lane and S. Eilenberg “General Theory of Natural Equivalences,” which was presented to the American Mathematical Society on September 8, 1942 and published in 1945 in the *Transactions of the AMS*.

Mac Lane authored or coauthored more than 100 research papers and 6 books: A SURVEY OF MODERN ALGEBRA (1941, 1997; with G. Birkhoff); HOMOLOGY (1963); ALGEBRA (1967; with G. Birkhoff); CATEGORIES FOR THE WORKING MATHEMATICIAN (1971); MATHEMATICS, FORM AND FUNCTION (1985); SHEAVES IN GEOMETRY AND LOGIC: A FIRST INTRODUCTION TO TOPOS THEORY (1992; with Ieke Moerdijk).

Mac Lane was the advisor of 39 Ph.D. theses. Alfred Putman, John Thompson, Irving Kaplansky, Robert Solovay, and many other distinguished scientists are listed as his students. He was elected to the National Academy of Sciences of the USA in 1949 and

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received the National Medal of Science, the highest scientific award of the USA in 1989. Mac Lane served as vice-president of the National Academy of Sciences and the American Philosophical Society. He was elected as president of the American Mathematical Society and Mathematical Association of America. He contributed greatly to modernization of the teaching programs in mathematics. Mac Lane received many signs of honor from the leading universities of the world and possessed an impressive collection of mathematical awards and prizes. Mac Lane became a living legend of the science of the USA.

Mac Lane was born on August 4, 1909 in Norwich near Taftville, Connecticut in the family of a Congregationalist minister and was christened as Leslie Saunders MacLane. The name Leslie was suggested by his nurse, but his mother disliked the name. A month later, his father put a hand on the head of the son, looked up to the God, and said: “Leslie forget.” His father and uncles changed the spelling of their surname and began to write MacLane instead of MacLean in order to avoid sounding Irish. The space in Mac Lane was added by Saunders himself at request of his first wife Dorothy. That is how Mac Lane narrated about his name in *A Mathematical Biography* which was published soon after his death.

Saunders’s father passed away when the boy was 15 and it was Uncle John who supported the boy and paid for his education in Yale. Saunders was firstly fond of chemistry but everything changed after acquaintance with differential and integral calculus by the textbook of Longley and Wilson (which reminds of the later book by Granville, Smith, and Longley). The university years revealed Mac Lane’s attraction to philosophy and foundations of mathematics. He was greatly impressed by the brand-new three volumes by Whitehead and Russell, the celebrated PRINCIPIA MATHEMATICA. The mathematical tastes of Mac Lane were strongly influenced by the lectures of a young assistant professor Oystein Ore, a Norwegian mathematician from the Emmy Noether’s school. After graduation from Yale, Mac Lane continued education in the University of Chicago. At that time he was very much influenced by the personalities and research of Eliakim Moore, Leonard Dickson, Gilbert Bliss, Edmund Landau, Marston Morse, and many others. Mac Lane was inclined to write a Ph.D. thesis in logic but this was impossible in Chicago and so Saunders decided to continue education in Göttingen.

The stay in Germany in 1931–1933 was decisive for the maturity of Mac Lane’s gift and personality. Although David Hilbert had retired, he still delivered weekly lectures on philosophy and relevant general issues. The successor of Hilbert was Hermann Weyl who had recently arrived from Zürich and was in the prime of his years and talents. Weyl advised Saunders to attend the lectures on linear associative algebras by Emmy Noether whom Weyl called “the equal of each of us.” In the Mathematical Institute Mac Lane met and boiled with Edmund Landau, Richard Courant, Gustav Herglotz, Otto Neugebauer, Oswald Teichmüller, and many others. Paul Bernays became the advisor of Mac Lane’s Ph.D. thesis “Abbreviated Proofs in Logic Calculus.”

The Nazis gained power in Germany in February 1933. The feast of antisemitism started immediately and one of the first and fiercest stroke fell upon the Mathematical Institute. The young persons are welcome to read as an antidote Mac Lane’s masterpiece “Mathematics at Göttingen under the Nazis” in *Notices of the AMS*, **42**:10, 1134–1138 (1995).

In the fall of 1933 Mac Lane returned to the States with Dorothy Jones Mac Lane whom he had married recently in Germany. The further academic career of Mac Lane was mainly tied with Harvard and since 1947 with Chicago.

To evaluate the contribution of Mac Lane to mathematics is an easy and pleasant task. It suffices to cite the words A. G. Kurosh, a renowned Russian professor of Lomonosov State University. In the translator’s preface to the Russian edition of the classical *Homology* book, Kurosh wrote:

The author of this book, a professor of Chicago University, is one the most prominent American algebraists and topologists. His role in homological algebra as well as category theory is the role of one of the founders of this area.

Homological algebra implements a marvelous project of algebraization of topological spaces by assigning to such a space X the sequence of (abelian) homology groups $H_n(X)$. Moreover, each continuous map $f : X \rightarrow Y$ from X to Y induces a family of homomorphisms of the homology groups $f_n : H_n(X) \rightarrow H_n(Y)$. The aim of homological algebra consists in calculation of homologies.

In his research into homological algebra and category theory Mac Lane cooperated with Eilenberg whom he met in 1940. Eilenberg had arrived from Poland two years earlier. He saw the affinity of the algebraic calculations of Mac Lane with those he encountered in algebraic topology. Eilenberg offered cooperation to Mac Lane. The union of Eilenberg and Mac Lane lasted for 14 years and resulted in 15 joint papers which noticeably changed the mathematical appearance of the 20th century.

The pearl of this cooperation was category theory. Mac Lane always considered category theory “a natural and perhaps inevitable aspect of the 20th century mathematical emphasis on axiomatic and abstract methods—especially as those methods when involved in abstract algebra and functional analysis.” he stressed that even if Eilenberg and he did not propose this theory it will necessarily appear in the works of other mathematicians. Among these potential inventors of the new conceptions Mac Lane listed Claude Chevalley, Heinz Hopf, Norman Steenrod, Henri Cartan, Charles Ehresmann, and John von Neumann.

In Mac Lane’s opinion, the conceptions of category theory were close to the methodological principles of the project of Nicholas Bourbaki. Mac Lane was sympathetic with the project and was very close to joining in but this never happened (the main obstacles were in linguistic facilities). However, even the later membership of Eilenberg in the Bourbaki group could not overcome a shade of slight disinclination and repulsion. It turned out impossible to “categorize Bourbaki” with a theory of non-French origin as Mac Lane had once phrased the matter shrewdly and elegantly. It is worth noting in this respect that the term “category theory” had roots in the mutual interest of its authors in philosophy and, in particular, in the works of Immanuel Kant.

Set theory rules in the present-day mathematics. The buffoon’s role of “abstract nonsense” is assigned in mathematics to category theory. History and literature demonstrate to us that the relations between the ruler and the jester may be totally intricate and unpredictable. Something very similar transpires in the interrelations of set theory and category theory and the dependency of one of them on the other.

From a logic standpoint, set theory and category theory are instances of a first order theory. The former deals with sets and the membership relation between them. The latter speaks of objects and morphisms (or arrows). Of course, there is no principle difference between the atomic formulas $a \in b$ and $a \rightarrow b$. However, the precipice in meaning is abysmal between the two concepts that are formalized by the two atomic formulas. The stationary universe of Zermelo-Fraenkel, cluttered up with uncountably many copies of equipollent sets confronts the free world of categories, ensembles of arbitrary nature that are determined by the dynamics of their transformations.

The individual dualities of set theory, dependent on the choice of particular realizations of the pair of objects under study, give up their places to the universal *natural transformations* of category theory. One of the most brilliant achievements of category theory was the development of axiomatic homology theory. Instead of the homological diversity for topological spaces (the simplicial homology for a polyhedron, singular and Čech homology, Vietoris homology, etc.) Eilenberg and Steenrod suggested as far back as in 1952 the new

understanding of each homology or cohomology theory as a functor from the category of spaces under consideration to the category of groups. The axiomatic approach to defining such a functor radically changed the manner of further progress in homological algebra and algebraic topology. The study of the homology of Eilenberg–Mac Lane spaces and the method of acyclic models demonstrated the strength of the ideas of category theory and led to universal use of simplicial sets in K -theory and sheaves.

In 1948 Mac Lane proposed the concept of abelian category abstracting the categories of abelian groups and vector spaces which played key roles in the first papers on axiomatic homology theory. The abelian categories were rediscovered in 1953 and became a major tool in research into homological algebra by Cartan, Eilenberg, and their followers.

Outstanding advances in category theory are connected with the names of Alexander Grothendieck and F. William Lawvere. Topos theory they created appeared in the course of “point elimination” called upon by the challenge of invariance of the object we study in mathematics. It is on this road that we met the conception of varying sets which led to the notion of topos and the understanding of the social medium of set-theoretic models.

A category is called an *elementary topos* provided that it is cartesian closed and has subobject classifier. The sources of toposes lie in the theory of sheaves and Grothendieck topology. Further progress of the concept of topos is due to search for some category-theoretic axiomatization of set theory as well as study into forcing and the nonstandard set-theoretic models of Dana Scott, Robert Solovay, and Petr Vopěnka. The new frameworks provide a natural place for the Boolean valued models that are viewed now the toposes with Aristotle logic which paved king’s ways to the solution of the continuum problem by Kurt Gödel and Paul Cohen. These toposes are now the main arena of Boolean valued analysis.

Bidding farewell to Mac Lane, reading his sincere and openhearted autobiography, enjoying his vehement polemics with Freeman J. Dyson, and perusing his deep last articles on general mathematics, anyone cannot help but share his juvenile devotion and love of mathematics and its creators. His brilliant essays “Despite Physicists, Proof Is Essential in Mathematics” and “Proof, Truth, and Confusion” form an anthem of mathematics which is only possible by proof.

Let me summarize where we have come. As with any branch of learning, the real substance of mathematics resides in the ideas. The ideas of mathematics are those which can be formalized and which have been developed to fit issues arising in science or in human activity. Truth in mathematics is approached by way of proof in formalized systems. However, because of the paradoxical kinds of self-reference exhibited by the barn door and Kurt Gödel, there can be no single formal system which subsumes all mathematical proof. To boot, the older dogmas that “everything is logic” or “everything is a set” now have competition—“everything is a function.” However, such questions of foundation are but a very small part of mathematical activity, which continues to try to combine the right ideas to attack substantive problems. Of these I have touched on only a few examples: Finding all simple groups, putting groups together by extension, and characterizing spheres by their connectivity. In such cases, subtle ideas, fitted by hand to the problem, can lead to triumph.

Numerical and mathematical methods can be used for practical problems. However, because of political pressures, the desire for compromise, or the simple desire for more publication, formal ideas may be applied in practical cases where the ideas simply do not fit. Then confusion arises—whether from misleading formulation of questions in opinion surveys, from nebulous calculations of airy benefits, by regression, by extrapolation, or otherwise. As the case of fuzzy sets indicates, such confusion is not fundamentally a trouble caused by the organizations issuing reports, but is occasioned by academicians making careless use of good ideas where they do not fit.

As Francis Bacon once said, “Truth ariseth more readily from error than from confusion.”

There remains to us, then, the pursuit of truth, by way of proof, the concatenation of those ideas which fit, and the beauty which results when they do fit.

So wrote Saunders Mac Lane, a great genius, creator, master, and servant of mathematics. His unswerving devotion to the ideals of truth and free thinking of our ancient science made him the eternal and tragicomical mathematical Knight of the Sorrowful Figure and Category...

For many years Saunders Mac Lane was an Honorary Editor of *Scientiae Mathematicae Japonicae*. It was a pleasure and honor for every member of the Editorial Board to be listed together with such an outstanding scholar and personality.