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Ú. L. Éršov and É. A. Palútin. ***Matématičéskaá logika*** (Mathematical logic). "Nauka," Moscow 1979, 320 pp.

This book was written for use both as a textbook for the standard university course in mathematical logic and as an aid for advanced graduate courses. It is based on lecture courses given at Novosibirsk University and is in fact a considerably extended version of the 159-page book with the same title by the same authors and M. A. Taitslin, published at Novosibirsk in 1973. Added are chapters on proof theory and recursion theory, and considerable material in model theory. A great deal of thought was obviously given both to the choice of material and to its presentation, but the haste with which the present version appears to have been produced and the elimination of the pedagogical influence of the third author have had a devastating effect: The book is full of defects of varying seriousness culminating in Chapter 6 where most of the proofs of standard theorems contain serious errors, and one of the important lemmas is simply false. Let us first review the material covered.

Since the course of logic is given during the first year in the universities of the USSR, the book is made to be self-contained. The first chapter (Propositional calculus) begins with the elementary properties of Boolean operations on sets. Then a natural deduction version of the classical propositional calculus is treated in detail up to normal forms of formulas, the completeness theorem, and the independence of rules. A more standard (Hilbert-type) formulation is also presented, and conservativity of (functionally complete) fragments is proved. Instead of establishing a normalization theorem for the natural deduction calculus, the authors introduce Gentzen's L-type formulation (with introduction rules both in the antecedent and in the succedent) and prove the cut-elimination theorem following Schütte.

Chapter 2 (Set theory) contains the necessary material on maps and relations, partially ordered sets, filters, ordinals and cardinals, and the axiom of choice. The authors reserve the term "model" for algebraic systems satisfying some set of predicate formulas. Basic model-theoretic notions are defined in Chapter 3 (Truth in algebraic systems), and the compactness theorem is proved via ultraproducts. In Chapter 4 (Predicate calculus), a natural deduction system for predicate logic with function symbols and equality is formulated (in such a way that the normal form theorem for derivations is not available), in addition to Hilbert-type systems. Elementary proof theory is developed up to the prenex normal form theorem (with atomic subformulas containing terms only of total degree $\leqq 1$), and the completeness theorem is established by extending a finite consistent set X to a maximal set containing along with any formula $\exists x F(x)$ the formula $F(c)$ for some c. The standard replacement of equality by a new predicate symbol with suitable axioms is presented, but to prove the conservativity of adding equality, the authors have to repeat the proof of the completeness theorem for the equality-free case.

Chapter 5 (Model theory) contains less elementary material: elementary equivalence (including the upward and downward Skolem theorems) and axiomatizable classes (including $\forall$- and $\exists$-axiomatizability, varieties and quasivarieties, and Skolem functions). The notion of a consistency mechanism allows the authors to isolate the common part in several model-theoretic constructions, including omitting-types and interpolation theorems. Proofs of (now) standard results on homogeneous, universal, and saturated models and categoricity are presented, and are supplemented by exposition of the second author's result: Every countably categorical quasivariety is categorical in any power $\neq 1$.

Chapter 6 (Proof theory) begins with an L-type (antecedent-succedent) formulation of the predicate calculus (without equality) admitting cut-elimination, but the proof of cut-elimination given here is based on a false inversion lemma (Proposition 3): If $F \rightarrow \exists x A(x)$ is provable then $F \rightarrow A(t_1) \vee \ldots \vee A(t_k)$ is provable for some $t_1, \ldots, t_k$. (Take $F = \exists x(p \,\&\, A(x))$.) The "proof" of this lemma is based on a false Lemma 3 providing the pure variable property as defined in the book: Eigenvariables occur only in predecessors of side formulas of corresponding rules (take the counterexample above). The proof of the second main result, the Herbrand theorem, also uses the false Proposition 3, and this time it cannot be repaired simply by replacing the defective parts by the corresponding parts of a standard proof. Finally, the completeness proof of (what the authors call) the calculus of resolvents is correct, but the formulation fails to mention unification, which is one of the most important features of the resolution calculus.

Chapter 7 (Algorithms and recursive functions) begins with some examples and elementary properties of Markov's normal algorithms and Turing machines. Then partial recursive functions are introduced via substitution, primitive recursion, and minimalization operators, and the possibility of using only (total) recursive functions in the definition of a recursive function is proved. The proof contains a gap in the induction step (of an induction on the length of the definition), and the most reasonable way to fill the gap seems to be to prove first the recursive enumerability of the graphs of partial recursive functions, established in the following section.

The normal form theorm for recursive functions is not explicitly formulated, but the existence of a universal recursively enumerable predicate and of a universal function is established. The recursive undecidability of the predicate calculus and Gödel's first incompleteness theorem are proved by using numeralwise representability of recursive functions in the extension of R. Robinson's system obtained by adding $x = y \vee x < y \vee x > y$. Gödel's second incompleteness theorem is not even formulated. In general the authors seem to feel more comfortable in the areas of model and recursion theory, where they have made contributions, than in proof theory. The essence of Hilbert's program, for example, is formulated in the introduction as "an attempt to construct such a formalization of mathematics, that by means of this system one can prove its own consistency." One has only to recall in this connection how many times Hilbert stressed the contentual character of his metamathematics as opposed to formal mathematics.

Much more surprising is the failure of the authors to use the apparatus they developed to give unified and manageable proofs of (many) proof-theoretic and (some) model-theoretic results. This point concerns mainly Gentzen-type calculi. The completeness theorem and the admissibility of cut (the normal form theorem for derivations) are most easily proved with the help of the universal proof search tree of a formula. As an additional bonus one has in this case Kreisel's correspondence between the paths (of given complexity) in such a tree with cut and the counter-models (of the same complexity) for the formula in question, which allows one to make a bridge to the senior author's topic of constructive models. Yet one finds in this book the indirect proof of the interpolation theorem containing, in fact, the direct construction using the normal form (not normalization) theorem for Gentzen-type derivations. If the authors had in mind to illustrate proof-theoretic transformations, the natural deduction system (used in this book only to ensure "The practical possibility of presenting all necessary formal proofs") is much more appropriate. In this case, of course, one has to stress the difference between normal form and normalization theorems.

To sum up, the book can be very useful as an aid for a lecturer in model theory, but not as a textbook. Both the notation and the formulations are too heavy for a beginner. The reader is invited to read the beginning of §24. Even a person with some experience should be constantly on guard against such surprises as an incorrect definition of the diagram of a model (p. 170) or the failure of induction in the proof of a model construction theorem (p. 188; formula $\Phi\neg$ can be longer than $\neg\Phi$). The reviewer was puzzled by a remark on page 167 on the "finitarity" of the notion of elementary equivalence. The proof one page 176 (of the condition for a formula to be an $\forall$-formula) obviously uses the distinctness of $b_1, \ldots, b_k$, which can be false. This list can be extended further.

It should be added that in the English translation of this book (see the following review), most of the serious errors—especially in Chapter 6—have been corrected. G. MINC

YU. L. ERSHOV and E. A. PALYUTIN. ***Mathematical logic***. Revised English translation by Vladimir Shokurov of the preceding. Mir Publishers, Moscow 1984, 303 pp.

The review above of the original Russian edition gives an adequate picture of the content of the English translation; note that the major errors have been corrected. The translator, however, has done a poor job. There are too many passages like the following: "The mechanism of compatibility is mainly of methodical importance. It allows us to recognize a common part in many theorems proving which involves construction of models." Moreover, since the Russian language has no indefinite and definite articles, their proper insertion in an English translation requires knowledge of the subject matter, something which the translator fails to display. Thus, this book is unsuitable for beginners. An experienced graduate student or logician will be able to glean interesting tidbits here and there. Especially valuable is Chapter 5 (*Model theory*), which contains concepts and results not found in other textbooks.

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