One General Method in Operator Theory

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A linear functional on a vector space is determined up to a scalar from its zero hyperplane. In contrast, a linear operator is recovered from its kernel up to a simple multiplier on a rather special occasion. Fortunately, Boolean valued analysis [1] prompts us that some operator analog of the functional case is valid for each operator with target a Kantorovich space, a Dedekind complete vector lattice. Indeed, we have the following descent of the Sard theorem.

Theorem 1. *Let Y be a universally complete Kantorovich space with base B and let S and T be order bounded linear operators from a Riesz space X to Y. Then $\ker(bS) \supset \ker(bT)$ for all $b \in B$ if and only if there is an orthomorphism α on Y such that $S = \alpha T$.*

Given $b \in B$, we call bT a *stratum* of T. It turns out that some properties of T are recovered from the properties of the kernels of the strata of T (cp. [2,3]).

Theorem 2. *An order bounded operator $T : X \to Y$ from a Riesz space X to a Kantorovich space Y is a difference of Riesz homomorphisms if and only if the kernel of every stratum bT of T with $b \in B$ is a Riesz subspace of X.*

Theorem 3. *The modulus of an order bounded operator $T : X \to Y$ is the sum of some pair of Riesz homomorphisms if and only if the kernel of each stratum bT of T with $b \in B$ is a Grothendieck subspace of X.*

References

[1] Kusraev A. G. and Kutateladze S. S. *Boolean Valued Analysis*, Dordrecht: Kluwer Academic Publishers (1999).

[2] Kutateladze S. S. On differences of Riesz homomorphisms. Sibirsk. Mat. Zh., **46**:2 (2005).

[3] Kutateladze S. S. On Grothendieck subspaces. Sibirsk. Mat. Zh., **46**:3 (2005).