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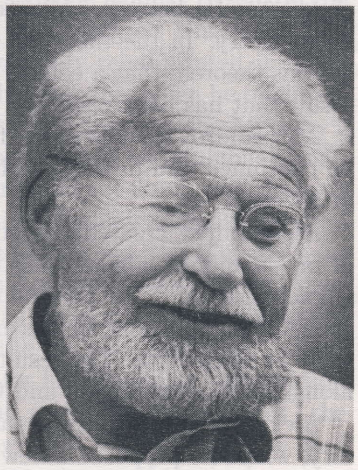
Aleksandr Danilovich Aleksandrov (obituary)



Academician Aleksandr Danilovich Aleksandrov had his 87th birthday on August 4, 1999. He was not able to celebrate it, because he died at 4:00 AM on July 27 in St. Petersburg. The world has lost an outstanding scientist and a brilliant man.

Aleksandr Danilovich was one of the leading mathematicians of the 20th century. His principal scientific achievements were in geometry. After Lobachevskii's discovery of non-Euclidean geometry and prior to the appearance of Aleksandrov's work there was only one world-class geometric discovery in Russia, E. S. Fedorov's construction of the theory of crystallographic groups, if one does not include as a part of geometry the topological investigations which were developed intensively in Russia in the early 20th century. In Aleksandrov, Russian science found a distinguished geometer who endowed this ancient science with a number of profound and original ideas. He made a great contribution to the development of the geometric conception of spaces, continuing the line started by Lobachevskii, Riemann, Cartan, and other leading geometers. Along with this it is necessary to stress that Aleksandrov was a mathematician with broad interests. He obtained important results in functional analysis, the theory of functions of a real variable, the theory of partial differential equations, mathematical crystallography, and other areas.

At the beginning of the 20th century geometry arrived at the study of objects "in the large". However, 19th-century methods of differential geometry and, a fortiori, 19th-century methods of studying the solubility of the Cauchy problem and boundary value problems for partial differential equations, did not give approaches to solving these problems. Only isolated results were obtained by the efforts of such important mathematicians as Minkowski, Hilbert, Weyl, and others. But their works contained the statements of many important unsolved problems which have determined the development of geometry "in the large" in this century. He achieved fundamental results in the study of these problems. Aleksandrov furthered the theory of mixed volumes, created by Minkowski. In particular, he proved a very general inequality for mixed volumes. His works on this theme reflect his profound geometric intuition and his mastery of the techniques of mathematical analysis. This stimulated the contemporary development of the connection between the theory of mixed volumes and the theory of functions of a complex variable. One of the results obtained by Aleksandrov turned out 50 years later to be a basic element in the solution of an old problem posed by van der Waerden.

Aleksandrov developed a theory of measure in abstract topological spaces and a geometric theory of weak convergence. This opened a way of introducing integral (not pointwise) functional characteristics in geometry and using weak convergence in the theory of ordinary and signed measures. His memoir on additive set functions in abstract spaces, published in *Matematicheskii Sbornik*, had great influence on the development of such parts of probability theory as limit theorems for stochastic processes. He became a pioneer in the application of functional-analytic methods in geometry. In the theory of functions of a real variable Aleksandrov proved his famous theorem that a convex function is twice differentiable almost everywhere, a result that has been re-proved many times by others.

Aleksandrov was one of the founders of the theory of non-regular curves, in which he continued and developed the ideas of such classical mathematicians as Jordan and Peano.

Aleksandrov created a special method that enables one to verify that a map of one manifold into another of the same dimension is actually a map onto the whole manifold. This method enabled him to prove a number of general theorems concerning conditions for the existence and uniqueness of a convex polyhedron with these or those data. The results of this series of papers placed Aleksandrov's name alongside those of Euclid and Cauchy.

Aleksandrov's most important results were obtained in the field of intrinsic geometry, where he proposed a new conception of space. Using approximation by polyhedra, he solved (in fact in a strong form, without any smoothness requirement) Weyl's problem on the realizability as a closed convex surface of every prescribed metric of non-negative curvature on the sphere. From the analytic point of view, he developed in these works a theory of generalized solutions for geometry, thus anticipating specialists in analysis and differential equations by several decades.

Aleksandrov used synthetic methods to study first the intrinsic geometry of convex surfaces and then the geometry of arbitrary two-dimensional manifolds of bounded curvature. The latter class, thanks to its compactness, serves as the space in which many extremal problems are solved. This class is in a way the closure of two-dimensional Riemannian manifolds.

In studying the intrinsic geometry of convex surfaces, Aleksandrov proved a "gluing theorem". It and the theorem on the realization of convex metrics form the basis of the modern theory of bending of convex surfaces with boundary in the class of convex manifolds.

For higher-dimensional metric spaces in which points are joined by shortest paths Aleksandrov introduced a general notion of the angle between shortest paths, and by comparing the angles of small triangles with the angles of a triangle with the same side-lengths on a two-dimensional surface of constant Gaussian curvature K he determined the spaces with curvature $\leq K$ and $\geq K$. They are now called "Aleksandrov spaces". In fact the rapid development of modern Riemannian geometry "in the large" began with Aleksandrov's theorem on the comparison of angles.

His works on the intrinsic geometry of metric manifolds place Aleksandrov's name alongside those of Gauss, Lobachevskii, and Riemann.

At the end of the 1950s Aleksandrov completed a series of papers devoted to uniqueness theorems and estimates of solutions for partial differential equations of elliptic type. Both the statement of the problem and the method of solution in each of these papers are of geometric origin.

We should also point out the large corpus of papers by Aleksandrov and his students on chronogeometry, the foundations of relativity theory. Aleksandrov is justifiably regarded as one of the founders of this direction of modern geometry.

For a twelve-year period, from 1952 to 1964, Aleksandr Danilovich was Rector of Leningrad (now St. Petersburg) State University. As Rector, Aleksandrov energetically supported the university biologists in their struggle against Lysenko-type pseudo-science. The teaching of scientific genetics at Leningrad University had begun in the 1950s, while at other universities it was only rehabilitated in 1965. This was very difficult: one need only recall Khrushchev's shouts when he qualified, as an exhibition of Menshevism, Aleksandrov's refusal to carry out the Ministry's order concerning the re-instatement at Leningrad University of an infamous obscurantist proponent of "Michurinite" biology. Aleksandrov did not flinch, and the man was not hired at Leningrad University. At the same time, biology students who had been dismissed from other universities for attempts to study genetics illegally were given the opportunity to continue their education in Leningrad. As Rector, Aleksandrov also established such new directions of study as sociology and mathematical economics, which received his active support within the walls of Leningrad University during a time of persecution. In October, 1990, for his particular contribution to the preservation and development of genetics and selection, Aleksandrov, the only mathematician in a group of biologists, was awarded the Order of the Red Banner of Labour. This unusual decoration was a consequence of the high esteem for Aleksandrov's noble action, given by an enormous majority of the scientific community of this country.

Aleksandr Danilovich spent 25 years of his life in Siberia. At the invitation of M. A. Lavrent'ev he moved in 1964 to Novosibirsk with his family. Here he found many true friends and students. He devoted not only his heart and soul to Siberia, but also his health, as a carrier of tick-borne encephalitis. Aleksandrov was the creator of a large and extensive school of research. There are dozens of recipients of advanced degrees among his students. As a research advisor he was distinguished by his attentiveness and by the generosity with which he shared ideas with his students.

Mathematics did not exhaust Aleksandrov's interests. He wrote many profound studies on questions of the philosophy and methodology of science. Aleksandr Danilovich was an outstanding sportsman: he received the title of master of sport in mountaineering.

Aleksandrov was awarded many decorations and distinctions. The very last one was the first Euler Gold Medal, awarded by the Presidium of the Russian Academy of Sciences in 1992.

Aleksandrov's scientific ideas will live for a long time in the work of his students and successors. The unique charm, the combination of youthful spirit and experienced wisdom, fierce temperament and subtle intellect, the selflessness and tenderness of Aleksandr Danilovich remain happy memories of all those who had the good fortune to be with him.

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[294] with S. S. Kutateladze and P. S. Filatov, All-Union Conference on Geometry and Analysis, *Uspekhi Mat. Nauk* **45**:3 (1990), 211–212. (Russian)

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[296] "If I were to give a lesson...", *Evrika* **1993**, no. 2. (Russian)

[297] "On Lobachevskiian geometry", *Mat. v Shkole* **1993**, no. 2, 2–7; no. 3, 2–5. (Russian)

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[1]This list continues those in *Uspekhi Mat. Nauk* **17**:6 (1962), 179–183 = *Russian Math. Surveys* **17**:6 (1962), 134–141; *ibid.* **28**:2 (1973), 252–253 = *ibid.* **28**:2 (1973), 228–230; *ibid.* **43**:2 (1988), 165–167 = *ibid.* **43**:2 (1988), 196–199; *ibid.* **48**:4 (1993), 239–241 = *ibid.* **48**:4 (1993), 259–260.