

MATHEMATICAL LIFE

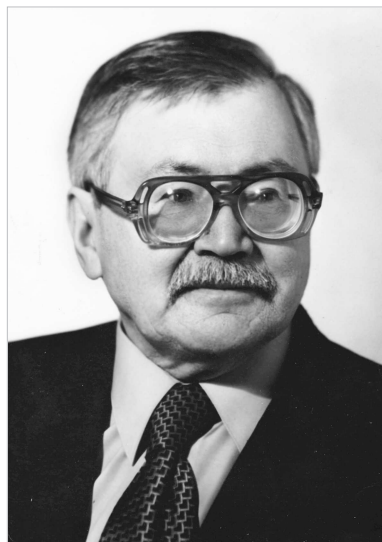
Yurii Grigor'evich Reshetnyak (on his 80th birthday)

On 26 September 2009 the prominent Russian scientist Academician Yu. G. Reshetnyak observed his 80th birthday.

The research interests of Reshetnyak cover an extremely wide range of problems in modern mathematics. Apart from the outstanding achievements relating to his first and main research interests—geometry and the theory of functions of a real variable, he has important results in mathematical physics, computational mathematics, functional analysis, and other areas of science bordering analysis and geometry. His creative work is characterized by exceptional depth, new and unexpected ideas, and original methods for penetrating to the essence of the problems under study.

Reshetnyak has fundamental results in geometry, the theory of functions, the classical calculus of variations, and a number of other areas. He is a founder of new directions in mathematics that lie on the border between analysis and geometry. One of these directions became known as the theory of spatial maps with bounded distortion (quasi-regular maps). The latter constitute a multidimensional real analogue of analytic functions and a ‘multivalent’ generalization of spatial quasi-conformal maps. Reshetnyak’s papers laid the foundations of non-linear potential theory connected with the notion of (l, p) -capacity. In the framework of this direction, substantial advances were achieved in the theory of functions with generalized derivatives. Reshetnyak’s results are a basis for the investigations of the school he created, which includes several dozen people with Ph.D. and D.Sc. degrees.

The authority of Siberian mathematics in the area of analysis and geometry is connected to a significant extent with the personal achievements of Reshetnyak, many of which have long ago become classical. First of all, we should mention here his celebrated theorem on isothermal coordinates on two-dimensional manifolds of bounded curvature, which were introduced by A. D. Aleksandrov. Reshetnyak’s definitive solution of M. A. Lavrent’ev’s problem on the stability of conformal maps has become known world-wide. His theorems on the weak convergence of Jacobians



and on the lower semicontinuity of functionals in the calculus of variations have become classical.

The research projects of Reshetnyak were realized in a number of monographs and more than two hundred papers. During his pedagogical activities he wrote more than 20 textbooks. Great meticulousness and legendary efficiency are distinctions of his scientific creative work.

It is hard to overestimate his contributions to the education of the next generation of researchers. He has devoted many years to the Faculty of Mechanics and Mathematics at Novosibirsk University, where he is head of the Department of Mathematical Analysis. His activities connected with the establishment and improvement of a modern course in mathematical analysis, which can quite rightly be called selfless, have to a great extent promoted the conception of teaching at a young university which has quickly gained a solid reputation as a high-class centre for the training of mathematicians. His lecture notes and his numerous textbooks on modern branches of analysis and on difficult chapters of the basic course have enjoyed the popularity of students and lecturers now for more than forty years, both at Novosibirsk University and at other leading universities of the country. We should also underscore the scientific generosity that is characteristic of Reshetnyak. Many of his ideas have been the bases for papers and dissertations of his students, and have determined their further creative activities.

Reshetnyak has devoted a lot of effort to the creation, establishment, and formation of the scientific profile of the *Sibirskii Matematicheskii Zhurnal* (translated as *Siberian Mathematical Journal*), on which he has actively worked since the first days of its organization. In the fact that it has constantly had one of the highest ratings among our country's general mathematical journals there is a large personal contribution of Reshetnyak.

He is distinguished by his rare modesty, sensitivity and consideration of people, tactfulness and reservedness in personal contacts, erudition, and mild humour. The style that is characteristic of the Leningrad–Petersburg mathematical school, which is realized by Reshetnyak in his everyday activities, has in no small measure helped young scientists and mathematicians of Siberia to form proper ideas about the principles of service to our country and about the ethical norms of a decent person.

Reshetnyak was born in Leningrad. In 1947, after graduation from secondary school, he entered the Faculty of Mathematics and Mechanics at Leningrad University. He completed his studies in four years and continued as a Ph.D. student at Leningrad University. His advisor was A. D. Aleksandrov. During his years of graduate studies the foundation was laid for the fruitful collaboration of Aleksandrov and Reshetnyak which continued for more than half a century, until Aleksandrov's death in 1999. In 1954 Reshetnyak defended his Ph.D. dissertation, "On the length and turn of a curve and on the area of a surface", and was sent to work in the Leningrad Division of the Steklov Mathematical Institute.

In 1957 a decision was taken for the creation of a new scientific establishment in the centre of Russia: the Siberian Division of the Academy of Sciences. Reshetnyak was among the first young researchers to respond to the call of the organizers of the Siberian Division — Academicians M. A. Lavrent'ev, S. L. Sobolev, and S. A. Khristianovich — and already by the end of 1957 had moved with his family to Novosibirsk, where he started to work at the new Institute of Mathematics. In Novosibirsk

he wrote all his main research works, and advanced along the difficult road from a young mathematician to a venerable academician. It was in Siberia that Reshetnyak's characteristic original style of investigations at the border between analysis and geometry finally took shape, his masterly and very distinctive mathematical techniques were created and perfected. In Novosibirsk in 1960, at the Joint Academic Council of the Siberian Division of the Academy of Sciences, he defended his doctoral dissertation, "Isothermal coordinates in two-dimensional manifolds of bounded curvature".

In the Sobolev Institute of Mathematics of the Siberian Division Reshetnyak created a research department which soon became the large Department of Analysis and Geometry. His scientific authority was so great that already in 1966, at the proposal of Academician A.I. Mal'tsev, he was elected head of the Department of Mathematical Analysis at Novosibirsk University, previously headed by M. A. Lavrent'ev and A. A. Lyapunov.

Reshetnyak has first-class achievements in the area of geometry. He made a fundamental contribution to the theory of manifolds of bounded curvature by proving his deep theorem on isothermal coordinates. This theorem states that a two-dimensional metric manifold has bounded curvature if and only if its metric element can be given in the form $ds^2 = \lambda(x, y)(dx^2 + dy^2)$, where the function $\log \lambda$ is the difference of two subharmonic functions. No other result of comparable power and generality exists in multidimensional Riemannian geometry up to the present day. This theorem made completely clear the connection between the central object of the modern theory of two-dimensional surfaces and the theory of functions. His isothermal coordinates made it possible to replace the ordinary synthetic methods of geometry 'in the large' by the purely analytic apparatus of quasi-conformal maps and quasi-linear equations of elliptic type.

Of course, two-dimensional manifolds of bounded curvature do not exhaust the entire range of geometric interests of Reshetnyak. The experts value greatly his results on boundedness of the turn of a shortest path, on the set of values of a countably additive vector-valued set function and on the mixing of closed intervals, his very subtle and deep results in integral geometry, and other results.

A significant number of his papers are devoted to the theory of functions of several real variables and its applications to geometry, functional analysis, differential equations, and so on.

As already mentioned, Reshetnyak gave the most complete solution of Lavrent'ev's problem on stability in Liouville's theorem on conformal maps of space. Solving this problem required developing fundamentally new approaches. Reshetnyak's investigations on the theory of quasi-conformal maps led him to the creation of new areas of analysis: the theory of maps with bounded distortion and related important aspects of non-linear potential theory.

He considered a wide range of questions that emerge in the study of spatial maps that have generalized derivatives in the sense of Sobolev, and he obtained a number of fundamental results in this direction. The synthesis of two directions, the classical theory of functions and the Sobolev function classes, proved to be so fruitful that it merited a special name: quasi-conformal analysis. This new direction is being actively developed in many countries.

In connection with investigations of the problem of stability in Liouville's theorem on conformal maps, Reshetnyak developed an interesting method of constructing integral representations of functions via values of differential operators. As an application of his method, he obtained estimates for a vector-valued function in terms of the corresponding deformation tensor (analogous estimates in mechanics are called Korn inequalities), and also in terms of the conformal deformation tensor.

The theory of non-linear capacity constructed by Reshetnyak is of significant interest in its own right. The original methods he found for describing the discontinuity sets of functions in Sobolev classes and for studying the behaviour of such functions near discontinuities were highly valued. He showed that the non-linear capacity he constructed by using Bessel potentials plays a role for Sobolev classes that is extremely similar to the role of Lebesgue measure, preserving (unlike the latter) information about the differentiability properties of functions. In each equivalence class of functions in the space W_p^l there exists a representative that is defined everywhere except for a set of zero capacity. These observations of Reshetnyak enabled him to find very subtle characteristics of the behaviour of functions with generalized derivatives. In particular, for functions specified more precisely with respect to their capacity, he proved theorems of Egorov and Luzin type in which the role of the measure is played by the capacity. He also established non-trivial connections between capacity and Hausdorff measure, and thereby made it possible to obtain geometric characteristics of sets of small capacity.

In recent years Reshetnyak, together with a large group of his students, has been conducting research in a new fundamental direction: the theory of maps with bounded distortion on Carnot–Carathéodory groups.

In 1996 the 2nd augmented and revised edition of the book *Stability theorems in geometry and analysis* appeared, published abroad by Kluwer Academic Publishers. In 1999–2001 Reshetnyak's 'four-book' *Course of mathematical analysis* finally appeared, on which he had worked for about forty years.

He defended his doctoral dissertation at the age of 31, and before the age of 33 he had already received the title of professor. In 1980 he was named an Honoured Scientist of the USSR. In 1981 he was elected a corresponding member of the Academy of Sciences, and in 1987 he became a full member. He was elected a foreign member of the Finnish Academy of Sciences in 1996, and an honorary member of the Moscow Mathematical Society in 1997. He has been awarded the "Badge of Honour" order, the medal of the order "For Service to the Fatherland" of second degree, and other medals.

In 2000 Reshetnyak's cycle of papers "Analytic studies of two-dimensional manifolds of bounded curvature" was distinguished with the Lobachevskii Prize of the Russian Academy of Sciences.

Investigations in the area of quasi-conformal analysis and non-linear potential theory are being intensively carried out all over the world. The significance of Reshetnyak's theory for applications to the theory of Sobolev spaces, for analysis of the boundary behaviour of functions of several complex variables and solutions of quasi-linear elliptic equations, and for applications in a number of other mathematical directions becomes more and more apparent.

Reshetnyak celebrates this anniversary in a circle of his loving close relatives, true friends, and talented students. New research and literary projects await him.

With all our hearts we sincerely wish Yurii Grigor'evich successes in his research, good health, and happiness in all its various manifestations!

*A. Yu. Vesnin, S. K. Vodop'yanov, S. S. Kutateladze,
S. P. Novikov, and I. A. Taimanov*

Translated by E. KHUKHRO