One Puzzling Definition and Model Theory

Kutateladze S. S.[1]

[1] *Sobolev Institute, Novosibirsk, Russia;* sskut@member.ams.org

1. Discretization is approximation of arbitrary function spaces and operators by their analogs in finite dimensions. Discretization matches the marvelous universal understanding of computational mathematics as the science of finite approximations to general (not necessarily metrizable) compacta. This revolutionary and challenging definition was given in the joint talk submitted by S. L. Sobolev, L. A. Lyusternik, and L. V. Kantorovich at the Third All-Union Mathematical Congress in 1956. Infinitesimal methods suggest a background, providing new schemes for discretization of general compact spaces. As an approximation to a compact space we may take an arbitrary internal subset containing all standard elements of the space under approximation.

2. Hypodiscretization of the equation $Tx = y$, with $T : X \to Y$ a bounded linear operator between some Banach spaces X and Y, consists in choosing finite-dimensional vector spaces X_N and Y_N, the corresponding embeddings $\imath_N$ and $\jmath_N$, and some operator $T_N : X_N \to Y_N$. In this event, the equation $T_N x_N = y_N$ is viewed as a finite-dimensional approximation to the original problem $Tx = y$.

3. Hyperdiscretization in contrast to hypodiscretization consists in approximating $Tx = y$ by the equation $T^{\#}x = y$, where $T^{\#} : X^{\#} \to Y^{\#}$ acts between external hyperfinite-dimensional spaces $X^{\#}$ and $Y^{\#}$ while $^{\#}$ symbolizes the taking of a nonstandard hull for spaces and operators.

4. Scalarization in the most general sense means reduction to numbers. Since each number is a measure of quantity, the idea of scalarization is clearly of a universal importance to mathematics. The deep roots of scalarization are revealed by Boolean valued models. Scalarization is effective in operator theory and multicriteria optimization.

5. Adaptation of the ideas of model theory to analysis projects among the synthetic methods of the present-day mathematics. This yields new models of numbers, spaces, and types of equations. The content expands of all available theorems and algorithms. The whole methodology of mathematical research is enriched and renewed, opening up absolutely fantastic opportunities. We can now use actual infinities and infinitesimals, transform matrices into numbers, spaces into straight lines, and noncompact spaces into compact spaces, yet having still uncharted vast territories of new knowledge. There is no backward traffic in science, and the new methods are doomed to reside in the realm of mathematics for ever and in a short time they will become as elementary and omnipresent in calculuses and calculations as Banach spaces and linear operators.

REFERENCES

1. *Sobolev S. L., Lyusternik L A., and Kantorovich L. V.,* "Functional Analysis and Computational Mathematics," in: Proceedings of the Third All-Union Mathematical Congress, Moscow, June–July 1956 (Moscow, 1956), Vol. 2, p. 43.
2. *Kusraev A. G. and Kutateladze S. S.,* Introduction to Boolean Valued Analysis, Nauka, Moscow (2005).
3. *Gordon E. I., Kusraev A. G., and Kutateladze S. S.,* Infinitesimal Analysis: Selected Topics. Nauka, Moscow (2008).