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# TWO APPLICATIONS OF LOGIC TO MATHEMATICS

*by*

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# Part I

## Boolean Valued Analysis

In 1963, Paul J. Cohen introduced the notion of forcing and proved many important independency results in set theory. In 1966, D. Scott and R. Solovay reformulated the theory of forcing in terms of Boolean valued models. (A similar idea was developed by Vopenka [12], [13].) We propose to apply the theory of Boolean valued models to analysis. We will introduce the idea of a Boolean valued model but without much explanation. However, the theory is easy to understand if one does several exercises after the basic properties of the model are stated. We will assume the basic properties of self-adjoint operators and their spectral decompositions.

D. Scott not only started Boolean valued models of set theory, with Solovay, but also introduced Boolean valued analysis. His choice of complete Boolean algebras was measure algebras. We will revisit Scott's Boolean valued analysis in Chapter 2.

The idea of Boolean valued models goes back to Church [4] and Rasiowa and Sikorski [8]. However it is fair to say that the success of Scott and Solovay's work has encouraged many similar developments and applications.

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