DYNAMIC EXTREMAL PROBLEMS AND INFINITESIMALS

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Abstract. The concept of infinitesimal optimality is specified for discrete dynamic extremal problems.

Recent research pays attention to approximate optimality dealing with extremal problems up to a prescribed accuracy level. This leads to the concept of epsilon-optimum and the corresponding calculus. In most cases it suffices to assume that this accuracy level is infinitesimal which simplifies the matter at the cost of invoking the nonstandard methods of analysis We set forth this idea in the case of discrete dynamic extremal problems.

Let $X_0,\dots,X_N$ be topological vector spaces, and let $G_k : X_{k-1} \rightrightarrows X_k$ be a nonempty convex correspondence for all $k := 1,\dots,N$. The collection $G_1,\dots,G_N$ determines the *dynamic family* of processes $(G_{k,l})_{k<l\le N}$, where the correspondence $G_{k,l} : X_k \rightrightarrows X_l$ is defined by the equalities

$$G_{k,l} := G_{k+1} \circ \dots \circ G_l \quad \text{if} \quad k+1<l;$$
$$G_{k,k+1} := G_{k+1} \quad (k := 0,1,\dots,N-1).$$

It is obvious that $G_{k,l} \circ G_{l,m} = G_{k,m}$ for all $k<l<m\le N$.

A *path* or *trajectory* of the above family of processes is defined to be an ordered collection of elements $\mathfrak{x} := (x_0,\dots,x_N)$ such that $x_l \in G_{k,l}(x_k)$ for all $k<l\le N$. Moreover, we say that x_0 is the beginning of $\mathfrak{x}$ and x_N is the ending of $\mathfrak{x}$.

Let E be a topological order complete vector lattice. Consider some convex operators $f_k : X_k \to E^{\bullet}$ $(k := 0,1,\dots,N)$ and convex sets $S_0 \subset X_0$ and $S_N \subset X_N$. Given a collection $\mathfrak{x} := (x_0,\dots,x_N)$, put

$$f(\mathfrak{x}) := \sum_{k=1}^{N} f_k(x_k).$$

A path is called *feasible* if its beginning belongs to S_0 and its ending, to S_N. A path $\mathfrak{x}^0 := \big(x_1^0,\dots,x_N^0\big)$ is called *infinitesimally optimal* if $x_0^0 \in S_0$, $x_N^0 \in S_N$, and $f(\mathfrak{x}^0)$ attains an infinitesimal minimum over the set of all feasible paths. This is an instance of a general *discrete dynamic extremal problem* which consists in finding a path of a dynamic family optimal in some sense. These problems are the topic of this talk.

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