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Foreword to the English Translation

I am deeply honored to introduce this great book of a great author to the English language reading community.

Denis Artem'evich Vladimirov (1929–1994) was a prominent representative of the Russian mathematical school in functional analysis which was founded by Leonid Vital'evich Kantorovich, a renowned mathematician and a Nobel Prize winner in economics.

This school comprises two affiliations in St. Petersburg and Novosibirsk which maintain intimate relations since the latter was set up by the former, so it is not astonishing that I enjoyed the wit and charm of Vladimirov for many years.

Our contacts were usually established through the students we supervised; he, in St. Petersburg and I, in Novosibirsk. I always tried to arrange matters so that my students spent some time near Vladimirov to master Boolean algebras and ordered vector spaces. Probably one of the results of this cooperation is the fact that there is now an active group in Boolean valued analysis in Novosibirsk. Unfortunately, the only possibility of continuing this practice is offered by the present book...

It was not long before Vladimirov's death when he and his friends had asked me to help with the publishing and editing of the English translation of the book. I agreed readily and soon Kluwer Academic Publishers decided to print the book.

The book was mostly translated by Professor A. E. Gutman and his students in Novosibirsk, all "descendants" of Vladimirov.

E. G. Taĭpale translated a few final sections and made the entire book more readable. I. I. Bazhenov, I. I. Kozhanova, Yu. N. Lovyagin, A. A. Samorodnitskiĭ, and Yu. V. Shergin helped me with the proofreading.

The translation took much more time than planned: the reasons behind this are understandable for anyone aware of the present standards of academic life in Russia. Unfortunately, capable mathematicians are not always experienced translators and knowledgeable grammarians. Therefore, the battle against solecism and mistranslation was partly lost in proofreading...

Vladimirov was unhappy that he had no opportunity to include a chapter on Boolean valued analysis in this edition of his book. At the publisher's request, I compiled a short appendix which is intended to serve as an introduction to this new and promising area for expansion and proliferation of Boolean algebras.

Denis Artem'evich Vladimirov was one of the giants of the past who bequeathed us his insight into part of the future with this book. I hope the reader will enjoy it.

S. S. Kutateladze

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