Contents

Foreword

The theory of vector lattices stemming from the mid-thirties undergoes the stage of summarizing achievements of the current stage of its development. The sweeping changes of the last two decades differ its image in a tremendous and complete fashion. The range of its application was expanded and enriched so as to embrace most diverse branches of the theory of functions, geometry of Banach spaces, operator theory, convex analysis, etc. Furthermore, the theory of vector lattices was impregnated with principally new tools and techniques from other sections of mathematics. These circumstances give rise to a row of monographs treating separate aspects of the theory and oriented to specialists. At the same time transparent becomes the necessity of a book intended for a wider readership whereas reflecting the modern directions of research. The present book is meant to be an attempt at implementing the task. Although oriented to the reader that makes the first acquaintance with vector-lattice theory, it is composed so that the main topics dealt with in the book reach the current level of research in the field, which is of interest and import for specialists.

The monograph was conceived so as to be divisible into two parts that can be read independently of one another. The first part is mainly Chapter 1 devoted to the so-called Boolean-valued analysis of vector lattices. The term designates the application of the theory of Boolean-valued models by D. Scott, R. Solovay and P. Vopěnka to constructing a special realization of (a model of) a vector lattice which allows one to treat elements of the lattice under study as reals. The starting point is an E. I. Gordon theorem claiming that the presentation of the field of reals in a Boolean-valued model constitutes a universally complete vector lattice (an extended K-space in the respective Russian terminology). Thus the huge part of the general theory of vector lattices admits of a straightforward derivation by interpreting the familiar properties of reals. The chapter also exposes Boolean-valued approaches to more advanced sections of vector-lattice theory such as lattice-normed spaces and fragments of positive operators, the approaches being proposed by A. G. Kusraev and S. S. Kutateladze. The elementary exposition of the apparatus of model theory as well as that of vector-lattice theory makes it possible to present the material in such a form that for a reader-logician it appears as an introduction into new fields of applications of model theory whereas for a reader-analyst it is an introduction into applicable model theory. More advanced topics of the abstract theory of operators in vector lattices are treated in Chapter 5 by A. E. Gutman. His research into the properties of disjointness preserving operators is in many aspects motivated by Boolean-valued analysis, demonstrating the power of the latter.

The second part of the monograph consists of Chapters 2–4 and a Supplement. It deals with operator theory in spaces of measurable functions and is oriented to the reader that is interested in functional analysis and the theory of functions. The book treats the classes of operators that are explicitly or implicitly tied with the natural order relation between measurable functions.

The explicit connection is discussed for instance in considering regular operators which are differences of pairs of positive operators. The implicit connection relates to integral operators whose definition is given in the conventional terms of function theory. It turns out that the answer to the question of what operators are integral leans upon the theory of vector lattices not only in formulation but also in proof. Part of Chapter 2 and the first part of Chapter 4 address the answering of the just-mention question first raised by John von Neumann as long ago as in the thirties. The book presents the original solution to the problem that was given by A. V. Bukhvalov (1974) and supplements it with the approaches that have appeared since then.

The theme of Chapter 3 and the second part of Chapter 4 is mainly related with studying stability of different classes of operators defined in terms of order; the stability is meant as regards multiplication by arbitrary continuous operators. As a rule it is relatively easy to demonstrate that such composition does not always belong to the class considered initially. Thus appears the problem of describing subclasses of operators stable under the indicated operation. In Chapter 3 we study various modifications of the problem for regular and dominated operators. The main results in this direction are due to B. M. Makarov and V. G. Samarskiĭ. The technique of researching the topic turns out to be interwoven with the theory of p-absolutely summing operators and operator factorization theory of E. M. Nikishin-B. Maurey. This material related to the modern Banach space theory is exposed in monographic form for the first time. The second part of Chapter 4 treats the same problem on composition but now for the class of integral operators.

In Chapter 4 the authors try their best to provide comprehensive information on solution of problems raised in the book of P. Halmos and V. Sunder "Bounded Linear Operators on L^2 Spaces." The main results here are due to the contribution of V. B. Korotkov and W. Schachermayer. In a separate supplement the book treats a related problem for the convolution operator which was settled by V. D. Stepanov.

The variety of addressed subjects and results determined the style of exposition. Part of a more elementary material freely accessible to the reader is presented without proofs. All principal results are however furnished with complete proofs. Commentaries appended to all chapters contain additional information and guide to the literature. While writing the book, the contributors assume the reader to be familiar with standard courses in the theory of functions and functional analysis.

The present collection is based on its predecessor in Russian which is enriched by Chapter 5 written by A. E. Gutman at my request. The Russian edition was a joint venture and a joint monograph by A. V. Bukhvalov, V. B. Korotkov, A. G. Kusraev, B. M. Makarov and S. S. Kutateladze which was published in 1992. Tumultuous events in the former Soviet Union hinder the means of communications between the contributors. As a result, I become the only one of our team who had a chance of reading the whole manuscript in English. So, I solely bear full responsibility for all demerits of the present edition, pretending to none of its possible merits.

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