

# Diophantine approximation with mixed powers of Piatetski-Shapiro primes

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## Abstract

Let  $[\cdot]$  denote the floor function. In this paper, we show that whenever  $\eta$  is real and the constants  $\lambda_i$  satisfy some necessary conditions, then for any fixed  $\frac{63}{64} < \gamma < 1$  and  $\theta > 0$ , there exist infinitely many prime triples  $p_1, p_2, p_3$  satisfying the inequality

$$|\lambda_1 p_1 + \lambda_2 p_2 + \lambda_3 p_3^2 + \eta| < (\max\{p_1, p_2, p_3^2\})^{\frac{63-64\gamma}{52} + \theta}$$

and such that  $p_i = [n_i^{1/\gamma}]$ ,  $i = 1, 2, 3$ .

**Keywords:** Diophantine approximation, Piatetski-Shapiro primes.

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## 1 Introduction and statement of the result

The study of Diophantine inequalities involving prime numbers constitutes a rapidly evolving field within analytic number theory. In 1967, A. Baker [1] proved that if  $\lambda_1, \lambda_2, \lambda_3$  are non-zero real numbers, not all of the same sign,  $\lambda_1/\lambda_2$  is irrational,  $\eta$  is real and  $A > 0$ , then there exist infinitely many prime triples  $p_1, p_2, p_3$  such that

$$|\lambda_1 p_1 + \lambda_2 p_2 + \lambda_3 p_3 + \eta| < \varepsilon_1, \tag{1}$$

where  $\varepsilon_1 = (\log \max p_j)^{-A}$ . Subsequently, the right-hand side of (1) was improved by Ramachandra [17], Vaughan [22], Lau and Liu [8], Baker and Harman [2] and Harman [7]. The best result to date is due to Matomäki [13], with  $\varepsilon_1 = (\max p_j)^{-\frac{2}{9} + \delta}$  and  $\delta > 0$ . In 2018, Gambini, Languasco and Zaccagnini [6] proved the existence of infinitely many triples of primes  $p_1, p_2, p_3$  such that

$$|\lambda_1 p_1 + \lambda_2 p_2 + \lambda_3 p_3^2 + \eta| < \varepsilon_2, \tag{2}$$

where  $\varepsilon_2 = (\max\{p_1, p_2, p_3^2\})^{-\frac{1}{12} + \delta}$  and  $\delta > 0$ . Weaker results were previously obtained in [9] and [14]. Another interesting question is the study of Diophantine inequalities

involving special prime numbers. Let  $P_l$  is a number with at most  $l$  prime factors. Very recently Todorova and Georgieva [21] solved inequality (2) with prime numbers  $p_1, p_2, p_3$  such that  $p_i + 2 = P_{l_i}, i = 1, 2, 3$ . In 1953, Piatetski-Shapiro [16] showed that for any fixed  $\frac{11}{12} < \gamma < 1$ , there exist infinitely many prime numbers of the form  $p = [n^{1/\gamma}]$ . Such primes are called Piatetski-Shapiro primes of type  $\gamma$ . Subsequently, the interval for  $\gamma$  was sharpened many times and the best result to date has been supplied by Rivat and Wu [19] with  $\frac{205}{243} < \gamma < 1$ . The primes of the form  $[n^{1/\gamma}]$  are very much in focus nowadays and many problems are solved using them. We mention, for example, papers [10], [11] and [12]. In 2022, the author [4] proved that for any fixed  $\frac{37}{38} < \gamma < 1$ , the inequality (1) is solvable with infinitely many Piatetski-Shapiro prime triples  $p_1, p_2, p_3$  of type  $\gamma$ . As a continuation of these studies, we solve (2) with Piatetski-Shapiro primes.

**Theorem 1.** *Suppose that  $\lambda_1, \lambda_2, \lambda_3$  are nonzero real numbers, not all of the same sign, that  $\lambda_1/\lambda_2$  is irrational, and that  $\eta$  is real. Let  $\theta > 0$  and  $\gamma$  be fixed with  $\frac{63}{64} < \gamma < 1$ . Then there exist infinitely many ordered triples of Piatetski-Shapiro primes  $p_1, p_2, p_3$  of type  $\gamma$  such that*

$$|\lambda_1 p_1 + \lambda_2 p_2 + \lambda_3 p_3^2 + \eta| < (\max\{p_1, p_2, p_3^2\})^{\frac{63-64\gamma}{52} + \theta}.$$

## 2 Notations

The letter  $p$  will always denote a prime number. By  $\delta$  we denote an arbitrarily small positive number, not the same in all appearances. As usual,  $[t]$  and  $\{t\}$  denote the integer part and the fractional part of  $t$ , respectively. Moreover  $\psi(t) = \{t\} - \frac{1}{2}$ . We write  $e(t) = e^{2\pi i t}$ . Let  $\gamma, \theta$  and  $\lambda_0$  be a real constants such that  $\frac{63}{64} < \gamma < 1, \theta > 0$  and  $0 < \lambda_0 < 1$ . Since  $\lambda_1/\lambda_2$  is irrational, there are infinitely many different convergents  $a_0/q_0$  to its continued fraction, with  $|\frac{\lambda_1}{\lambda_2} - \frac{a_0}{q_0}| < \frac{1}{q_0^2}, (a_0, q_0) = 1, a_0 \neq 0$  and  $q_0$  is arbitrary large. Denote

$$X = q_0^{\frac{13}{6}}; \tag{3}$$

$$\Delta = X^{-\frac{12}{13}} \log X; \tag{4}$$

$$\varepsilon = X^{\frac{63-64\gamma}{52} + \theta}; \tag{5}$$

$$H = \frac{\log^2 X}{\varepsilon}; \tag{6}$$

$$S_k(t) = \sum_{\substack{\lambda_0 X < p^k \leq X \\ p = [n^{1/\gamma}]} } p^{1-\gamma} e(tp^k) \log p, \quad k = 1, 2; \tag{7}$$

$$\Sigma(t) = \sum_{\lambda_0 X < p^2 \leq X} e(tp^2) \log p; \tag{8}$$

$$U(t) = \sum_{\lambda_0 X < n^2 \leq X} e(tn^2); \tag{9}$$

$$\Omega(t) = \sum_{\lambda_0 X < p^2 \leq X} p^{1-\gamma} (\psi(-(p+1)^\gamma) - \psi(-p^\gamma)) e(tp^2) \log p; \quad (10)$$

$$I_k(t) = \int_{(\lambda_0 X)^{\frac{1}{k}}}^{X^{\frac{1}{k}}} e(ty^k) dy, \quad k = 1, 2. \quad (11)$$

### 3 Preliminary lemmas

**Lemma 1.** *Let  $\varepsilon > 0$  and  $k \in \mathbb{N}$ . There exists a function  $\theta(y)$  which is  $k$  times continuously differentiable and such that*

$$\begin{aligned} \theta(y) &= 1 & \text{for } |y| &\leq 3\varepsilon/4; \\ 0 < \theta(y) &< 1 & \text{for } 3\varepsilon/4 < |y| < \varepsilon; \\ \theta(y) &= 0 & \text{for } |y| &\geq \varepsilon. \end{aligned}$$

and its Fourier transform

$$\Theta(x) = \int_{-\infty}^{\infty} \theta(y) e(-xy) dy$$

satisfies the inequality

$$|\Theta(x)| \leq \min \left( \frac{7\varepsilon}{4}, \frac{1}{\pi|x|}, \frac{1}{\pi|x|} \left( \frac{k}{2\pi|x|\varepsilon/8} \right)^k \right).$$

*Proof.* See ([15]). □

**Lemma 2.** *For any fixed  $\frac{2426}{2817} < \gamma < 1$ , we have*

$$\sum_{\substack{p \leq X \\ p = [n^{1/\gamma}]} } 1 \sim \frac{X^\gamma}{\log X}.$$

*Proof.* See ([18], Theorem 1). □

**Lemma 3.** *We have*

$$\begin{aligned} \text{(i)} \quad & \int_{-\Delta}^{\Delta} |S_1(t)|^2 dt \ll X \log^3 X, & \int_0^1 |S_1(t)|^2 dt \ll X^{2-\gamma} \log X, \\ \text{(ii)} \quad & \int_{-\Delta}^{\Delta} |I_1(t)|^2 dt \ll X, & \int_{-\Delta}^{\Delta} |I_2(t)|^2 dt \ll 1. \end{aligned}$$

*Proof.* For (i) see ([4], Lemma 6). For (ii) see ([21], Lemma 15). □

**Lemma 4.** *Let  $|t| \leq \Delta$ . Then the asymptotic formula*

$$S_1(t) = \gamma I_1(t) + \mathcal{O}\left(\frac{X}{e^{(\log X)^{1/5}}}\right)$$

*holds.*

*Proof.* See ([4], Lemma 5). □

**Lemma 5.** *Let  $\frac{13}{14} < \gamma < 1$ . Then*

$$\Omega(t) \ll X^{\frac{21-7\gamma}{29} + \delta}.$$

*Proof.* See ([5], Lemma 6). □

**Lemma 6.** *Let  $k \geq 1$  and  $1/2X \leq Y \leq 1/2X^{1-\frac{5}{6k}+\delta}$ . Then there exists a positive constant  $c_1(\delta)$ , which does not depend on  $k$ , such that*

$$\int_{-Y}^Y |\Sigma(t) - U(t)|^2 dt \ll \frac{X^{\frac{2}{k}-2} \log^2 X}{Y} + Y^2 X + X^{\frac{2}{k}-1} \exp\left(-c_1 \left(\frac{\log X}{\log \log X}\right)^{1/3}\right).$$

*Proof.* See ([6], Lemma 1 and Lemma 2). □

**Lemma 7.** *Let  $\frac{11}{12} < \gamma < 1$  and  $\Delta \leq |t| \leq H$ . Then there exists a sequence of real numbers  $X_1, X_2, \dots \rightarrow \infty$  such that*

$$\min \left\{ |S_1(\lambda_1 t)|, |S_1(\lambda_2 t)| \right\} \ll X_j^{\frac{37-12\gamma}{26}} \log^5 X_j, \quad j = 1, 2, \dots$$

*Proof.* See ([4], Lemma 7). □

**Lemma 8.** *We have*

$$\int_0^1 |S_2(t)|^4 dt \ll X^{2-\gamma+\delta}.$$

*Proof.* See ([23], (12)). □

## 4 Beginning of the proof

Consider the sum

$$\Gamma(X) = \sum_{\substack{\lambda_0 X < p_1, p_2, p_3^2 \leq X \\ p_i = [n_i^{1/\gamma}], i=1,2,3}} \theta(\lambda_1 p_1 + \lambda_2 p_2 + \lambda_3 p_3^2 + \eta) p_1^{1-\gamma} p_2^{1-\gamma} p_3^{1-\gamma} \log p_1 \log p_2 \log p_3. \quad (12)$$

Using the inverse Fourier transform for the function  $\theta(x)$ , we obtain

$$\begin{aligned} \Gamma(X) &= \sum_{\substack{\lambda_0 X < p_1, p_2, p_3^2 \leq X \\ p_i = [n_i^{1/\gamma}], i=1,2,3}} p_1^{1-\gamma} p_2^{1-\gamma} p_3^{1-\gamma} \log p_1 \log p_2 \log p_3 \\ &\quad \times \int_{-\infty}^{\infty} \Theta(t) e((\lambda_1 p_1 + \lambda_2 p_2 + \lambda_3 p_3^2 + \eta)t) dt \\ &= \int_{-\infty}^{\infty} \Theta(t) S_1(\lambda_1 t) S_1(\lambda_2 t) S_2(\lambda_3 t) e(\eta t) dt. \end{aligned}$$

We decompose  $\Gamma(X)$  as follows

$$\Gamma(X) = \Gamma_1(X) + \Gamma_2(X) + \Gamma_3(X), \quad (13)$$

where

$$\Gamma_1(X) = \int_{|t| < \Delta} \Theta(t) S_1(\lambda_1 t) S_1(\lambda_2 t) S_2(\lambda_3 t) e(\eta t) dt, \quad (14)$$

$$\Gamma_2(X) = \int_{\Delta \leq |t| \leq H} \Theta(t) S_1(\lambda_1 t) S_1(\lambda_2 t) S_2(\lambda_3 t) e(\eta t) dt, \quad (15)$$

$$\Gamma_3(X) = \int_{|t| > H} \Theta(t) S_1(\lambda_1 t) S_1(\lambda_1 t) S_2(\lambda_3 t) e(\eta t) dt. \quad (16)$$

We shall estimate  $\Gamma_1(X)$ ,  $\Gamma_2(X)$  and  $\Gamma_3(X)$ , respectively, in the Sections 5, 6 and 7. In Section 8 we shall complete the proof of Theorem 1.

## 5 Lower bound of $\Gamma_1(\mathbf{X})$

**Lemma 9.** *Let  $\frac{13}{14} < \gamma < 1$ . Then*

$$S_2(t) = \gamma \Sigma(t) + \mathcal{O}\left(X^{\frac{21-7\gamma}{29} + \delta}\right).$$

*Proof.* From (7), (8), (10) and the well-known asymptotic formula

$$(p+1)^\gamma - p^\gamma = \gamma p^{\gamma-1} + \mathcal{O}(p^{\gamma-2})$$

we write

$$\begin{aligned} S_2(t) &= \sum_{\lambda_0 X < p^2 \leq X} p^{1-\gamma} ([-p^\gamma] - [-(p+1)^\gamma]) e(tp^2) \log p \\ &= \sum_{\lambda_0 X < p^2 \leq X} p^{1-\gamma} ((p+1)^\gamma - p^\gamma) e(tp^2) \log p \\ &+ \sum_{\lambda_0 X < p^2 \leq X} p^{1-\gamma} (\psi(-(p+1)^\gamma) - \psi(-p^\gamma)) e(tp^2) \log p \\ &= \gamma \Sigma(t) + \Omega(t) + \mathcal{O}(1). \end{aligned} \tag{17}$$

Bearing in mind (17) and Lemma 5, we establish the statement in the lemma.  $\square$

Put

$$J(X) = \gamma^3 \int_{|t| < \Delta} \Theta(t) I_1(\lambda_1 t, X) I_1(\lambda_2 t, X) I_2(\lambda_3 t, X) e(\eta t) dt. \tag{18}$$

Now (4), (7), (8), (9), (11), (14), (18), Cauchy's inequality, Lemma 1, Lemma 3, Lemma 4 and Lemma 9 imply

$$\begin{aligned} \Gamma_1(X) - J(X) &= \gamma^2 \int_{|t| < \Delta} \Theta(t) (S_1(\lambda_1 t) - \gamma I_1(\lambda_1 t)) I_1(\lambda_2 t) I_2(\lambda_3 t) e(\eta t) dt \\ &+ \gamma \int_{|t| < \Delta} \Theta(t) S_1(\lambda_1 t) (S_1(\lambda_2 t) - \gamma I_1(\lambda_2 t)) I_2(\lambda_3 t) e(\eta t) dt \\ &+ \int_{|t| < \Delta} \Theta(t) S_1(\lambda_1 t) S_1(\lambda_2 t) (S_2(\lambda_3 t) - \gamma I_2(\lambda_3 t)) e(\eta t) dt \\ &\ll \varepsilon \frac{X}{e^{(\log X)^{1/5}}} \left[ \left( \int_{|t| < \Delta} |I_1(\lambda_2 t)|^2 dt \right)^{\frac{1}{2}} \left( \int_{|t| < \Delta} |I_2(\lambda_3 t)|^2 dt \right)^{\frac{1}{2}} \right. \\ &\quad \left. + \left( \int_{|t| < \Delta} |S_1(\lambda_1 t)|^2 dt \right)^{\frac{1}{2}} \left( \int_{|t| < \Delta} |I_2(\lambda_3 t)|^2 dt \right)^{\frac{1}{2}} \right] \\ &+ \varepsilon \int_{|t| < \Delta} |S_1(\lambda_1 t)| |S_1(\lambda_2 t)| \left| \gamma \Sigma(\lambda_3 t) - \gamma I_2(\lambda_3 t) + \mathcal{O}(X^{\frac{21-7\gamma}{29} + \delta}) \right| dt \end{aligned}$$

$$\begin{aligned}
&\ll \varepsilon \frac{X^{\frac{3}{2}}}{e^{(\log X)^{1/6}}} + \varepsilon \int_{|t| < \Delta} |S_1(\lambda_1 t)| |S_1(\lambda_2 t)| |\Sigma(\lambda_3 t) - U(\lambda_3 t)| dt \\
&+ \varepsilon \int_{|t| < \Delta} |S_1(\lambda_1 t)| |S_1(\lambda_2 t)| |U(\lambda_3 t) - I_2(\lambda_3 t)| dt \\
&= \varepsilon \left( \frac{X^{\frac{3}{2}}}{e^{(\log X)^{1/6}}} + J_1 + J_2 \right), \tag{19}
\end{aligned}$$

say. Using Cauchy's inequality, Lemma 2, Lemma 3 and Lemma 6, we get

$$J_1 \ll X \left( \int_{-\Delta}^{\Delta} |S_1(\lambda_1 t)|^2 dt \right)^{\frac{1}{2}} \left( \int_{-\Delta}^{\Delta} |\Sigma(\lambda_3 t) - U(\lambda_3 t)|^2 dt \right)^{\frac{1}{2}} \ll \frac{X^{\frac{3}{2}}}{e^{(\log X)^{1/6}}}. \tag{20}$$

By Euler's summation formula, we have

$$I_2(t) - U(t) \ll 1 + |t|X. \tag{21}$$

From (21), Cauchy's inequality and Lemma 3, we derive

$$J_2 \ll (1 + \Delta X) \left( \int_{-\Delta}^{\Delta} |S_1(\lambda_1 t)|^2 dt \right)^{\frac{1}{2}} \left( \int_{-\Delta}^{\Delta} |S_1(\lambda_2 t)|^2 dt \right)^{\frac{1}{2}} \ll \Delta X^2 \log^3 X. \tag{22}$$

On the other hand for the integral defined by (18), we write

$$J(X) = B(X) + \Phi, \tag{23}$$

where

$$B(X) = \gamma^3 \int_{-\infty}^{\infty} \Theta(t) I_1(\lambda_1 t) I_1(\lambda_2 t) I_2(\lambda_3 t) e(\eta t) dt.$$

and

$$\Phi \ll \int_{\Delta}^{\infty} |\Theta(t)| |I_1(\lambda_1 t) I_1(\lambda_2 t) I_2(\lambda_3 t)| dt. \tag{24}$$

Arguing as in ([3], Lemma 4), we deduce that if

$$\lambda_0 < \min \left( \frac{\lambda_1}{4|\lambda_3|}, \frac{\lambda_2}{4|\lambda_3|}, \frac{1}{16} \right)$$

then

$$B(X) \gg \varepsilon X^{\frac{3}{2}}. \quad (25)$$

By (11) and ([20], Lemma 4.2), we get

$$I_k(t) \ll X^{\frac{1}{k}-1} \min(X, |t|^{-1}). \quad (26)$$

Using (24), (26) and Lemma 1, we obtain

$$\Phi \ll \frac{\varepsilon X^{\frac{1}{2}}}{\Delta}. \quad (27)$$

Bearing in mind (4), (19), (20), (22), (23), (25) and (27), we establish

$$\Gamma_1(X) \gg \varepsilon X^{\frac{3}{2}}. \quad (28)$$

## 6 Upper bound of $\Gamma_2(\mathbf{X})$

Put

$$\mathfrak{S}(t, X) = \min \left\{ |S_1(\lambda_1 t)|, |S_1(\lambda_2 t)| \right\}. \quad (29)$$

Taking into account (15), (29), Lemma 1 and Lemma 7, we deduce

$$\begin{aligned} \Gamma_2(X_j) &\ll \varepsilon \int_{\Delta \leq |t| \leq H} \mathfrak{S}(t, X_j)^{\frac{1}{2}} |S_1(\lambda_1 t)|^{\frac{1}{2}} |S_1(\lambda_2 t)| |S_2(\lambda_3 t)| dt \\ &\quad + \varepsilon \int_{\Delta \leq |t| \leq H} \mathfrak{S}(t, X_j)^{\frac{1}{2}} |S_1(\lambda_1 t)| |S_1(\lambda_2 t)|^{\frac{1}{2}} |S_2(\lambda_3 t)| dt \\ &\ll \varepsilon X_j^{\frac{37-12\gamma}{52} + \delta} (\Psi_1 + \Psi_2), \end{aligned} \quad (30)$$

where

$$\Psi_1 = \int_{\Delta}^H |S_1(\lambda_1 t)|^{\frac{1}{2}} |S_1(\lambda_2 t)| |S_2(\lambda_3 t)| dt, \quad (31)$$

$$\Psi_2 = \int_{\Delta}^H |S_1(\lambda_1 t)| |S_1(\lambda_2 t)|^{\frac{1}{2}} |S_2(\lambda_3 t)| dt. \quad (32)$$

We estimate only  $\Psi_1$  and the estimation of  $\Psi_2$  proceeds in the same way. From (31) and Cauchy's inequality, we derive

$$\Psi_1 \ll \left( \int_{\Delta}^H |S_1(\lambda_2 t)|^2 dt \right)^{\frac{1}{2}} \left( \int_{\Delta}^H |S_1(\lambda_1 t)|^2 dt \right)^{\frac{1}{4}} \left( \int_{\Delta}^H |S_2(\lambda_3 t)|^4 dt \right)^{\frac{1}{4}}. \quad (33)$$

Using Lemma 3 (i), we obtain

$$\int_{\Delta}^H |S_1(\lambda_k t)|^2 dt \ll H X_j^{2-\gamma} \log X_j, \quad k = 1, 2. \quad (34)$$

By Lemma 8, we find

$$\int_{\Delta}^H |S_2(\lambda_3 t)|^4 dt \ll H X_j^{2-\gamma+\delta}. \quad (35)$$

Now (33) – (35) yield

$$\Psi_1 \ll H X_j^{2-\gamma+\delta}. \quad (36)$$

Combining (5), (6), (30) and (36), we get

$$\Gamma_2(X_j) \ll X_j^{\frac{37-12\gamma}{52}+\delta} X_j^{2-\gamma+\delta} = X_j^{\frac{141-64\gamma}{52}+\delta} \ll \frac{\varepsilon X_j^{\frac{3}{2}}}{\log X_j}. \quad (37)$$

## 7 Upper bound of $\Gamma_3(\mathbf{X})$

By (7), (16), Lemma 1 and Lemma 2, it follows

$$\Gamma_3(X) \ll X^3 \int_H^{\infty} \frac{1}{t} \left( \frac{k}{2\pi t \varepsilon / 8} \right)^k dt = \frac{X^3}{k} \left( \frac{4k}{\pi \varepsilon H} \right)^k. \quad (38)$$

Choosing  $k = \lceil \log X \rceil$  from (6) and (38), we deduce

$$\Gamma_3(X) \ll 1. \quad (39)$$

## 8 Proof of the Theorem

Summarizing (5), (13), (28), (37) and (39), we derive

$$\Gamma(X_j) \gg \varepsilon X_j^{\frac{3}{2}} = X_j^{\frac{141-64\gamma}{52}+\theta}.$$

The last estimation implies

$$\Gamma(X_j) \rightarrow \infty \quad \text{as} \quad X_j \rightarrow \infty. \quad (40)$$

Bearing in mind (12) and (40) we establish Theorem 1.

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