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A double VNS heuristic for the facility location and pricing problem

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Abstract

In this paper we present the problem of decision making on the facility location and pricing. We assume that the facilities can charge the different prices and the objective is to maximize the overall revenue. It is known that the problem is NP-hard in the strong sense even for the given facility location. A two level local search heuristic based on the VNS framework is developed for this nonlinear problem. To evaluate the global maximum, we reformulate the problem as a mixed integer linear program with additional constraints and variables. Computational results for randomly generated test instances are discussed.

Keywords: Local search, facility location, pricing, nonlinear optimization

1 Introduction

Facility location constitutes a broad spectrum of mathematical models, methods, and applications in operations research. In most part of the models we

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consider location – allocation aspects without pricing. Locations represent long–term decision, pricing represent short–term decisions [2]. As a results, we have to consider two stages approaches: first, location; later on, pricing [8]. Nevertheless, separation of location and pricing decisions may be not acceptable, for example, in the cases where locations are chosen conditionally on client's demand which, in turn, depends on prices [4]. Based on the pioneer paper of Hotelling [6], a lot of mathematical models for location and pricing decisions under competition have been studied (see, e.g., [2,1,3]). In this paper we assume that pre-existing facilities are fixed to their current prices and locations. They cannot easily adjust. A new company tries to open own facilities and charging own prices in order to maximize the revenue. In other words, we want to find the optimal strategy of the follower in a Stackelberg type leader–follower game [1]. We assume that the client demand is concentrated at a finite set of discrete points. Buying power of each client is assigned entirely to the facility providing the minimal sum of price and transportation cost. If this sum exceeds the corresponding value for the pre-existing facilities, then the client patronizes a leader facility.

There are three spatial pricing strategies identified by Hanjoul et al [4]:

— mill pricing, where each facility may charge a different price;
— uniform pricing, where facilities charge identical price;
— discriminatory pricing, where each client may be charged a different price.

In this paper we consider the mill pricing strategy, but the transportation cost may by different for the client of the same facility. Thus, we consider the following facility location and pricing problem. A company wishes to open some facilities and assigns prices for the product of each opening facility. Each client knows the transportation cost of servicing from each facility and has own budget (more precisely, this budget is a threshold defined by the leader facilities). Client selects a facility with minimal total payment: price and transportation cost. He buys the product if his payment does not exceed his budget. The objective is to find p facilities for the company and assign the prices for each opening facility in order to maximize the overall revenue.

In Section 2 we present exact mathematical model with nonlinear objective function and some nonlinear constraints. In Section 3 we describe our local search heuristics for this model. Since the pricing subproblem is NP-hard in the strong sense even for fixed location, we develop a two-level approach: local search for location and local search for pricing under fixed location. In such framework, the pricing subproblem has small dimension and we can solve it easily. For the pricing problem we use the VNS metaheuristic. For the location problem we apply the VNS and SA metaheuristics. In order to evaluate the quality of the solutions we reformulate the problem as a mixed integer program and apply the branch and bound method. In section 4 we discuss the computational results.

2 Mathematical model

We assume that
$J = \{1, \dots, n\}$ is the set of clients;
$I = \{1, \dots, m\}$ is the set of potential facility locations;
p is the number of opening facilities;
b_j is the budget of client j;
c_{ij} is the transportation cost for client j if he obtains the product of facility i.

Decision variables:
p_i is the price for the product of facility i;
$x_i = 1$ if facility i is opened and $x_i = 0$ otherwise;
$x_{ij} = 1$ if client j is serviced from facility i and $x_{ij} = 0$ otherwise.

Now we can present the problem as the following mixed integer nonlinear program

$$\max \sum_{i \in I} p_i \sum_{j \in J} x_{ij}$$

subject to

$$\sum_{i \in I} x_{ij} \leq 1, \quad j \in J;$$

$$x_i \geq x_{ij}, \quad i \in I, j \in J;$$

$$\sum_{i \in I} x_i = p;$$

$$\sum_{i \in I} (p_i + c_{ij}) x_{ij} \leq b_j, \quad j \in J;$$

$$(p_i + c_{ij}) x_{ij} \leq p_k + c_{kj}, \quad i, k \in I, i \neq k, \quad j \in J;$$

$$p_i \geq 0, \quad x_i, x_{ij} \in \{0, 1\}, \quad i \in I, j \in J.$$

The objective function is the total revenue of the company. The first constraint quarantines that each client selects at most one facility as supplier. The second constraint allows company to service clients from opened facilities only. The third constraint ensures that p facilities can be opened by the company. The fourth constraint is the budget constraint for each client. The fifth constraint describes the strategy of clients. Each client selects the cheapest variant according to the price and transportation cost.

3 Local search heuristics

This facility location and pricing problem is NP–hard in the strong sense. Moreover, it is NP–hard in the strong sense even for given facility location. Therefore, we develop a two-level heuristic:

— local search for facility location by the decision variables x_i,
— for the given p facilities, local search for the pricing problem by the decision variables p_i.

In this framework, the pricing problem has small dimension and we can evaluate the total revenue for the given location quickly. The VNS approach [5] is applied for this end. We use the neighborhoods N_k, $k = 1, \dots, k_{max}$, where the prices of at most k facilities are changed. For the facility location, we apply local search again but for other decision variables. SA and VNS heuristics are used in this stage of our method. The neighboring solutions are generated by the k-swap and Lin–Kernighan neighborhoods [7]. In the k-swap neighborhood we move at most k facilities to new locations. Surely, the finding of the best neighboring solution is time consuming procedure for large k. Thus, we use small k ($k \leq 3$) for the VNS and $k = 1$ for the SA.

The most interesting feature of our approach is the Lin–Kernighan neighborhood for the facility location. This neighborhood allows us to discover the most promising regions in the feasible domain. For given solution S which consists of p opened facilities, the neighboring solutions can be defined by the following way [7]:

1. Choose two facilities $i_{ins} \in I \setminus S$ and $i_{rem} \in S$ such that the objective function $F(S \cup \{i_{ins}\} \setminus \{i_{rem}\})$ is maximal even if it less than $F(S)$.
2. Perform exchange of i_{rem} and i_{ins}.
3. Repeat steps 1, 2 k' times so that a facility can not be chosen to be inserted in S if it has been removed from S in one of the previous iterations of step 1 and step 2.

The sequence $\{(i^{\tau}_{ins}, i^{\tau}_{rem})\}_{\tau \leq k'}$ defines k' neighboring solutions for S. We say that S is local maximum with respect to this neighborhood if $F(S) \geq F(S^\tau)$ for all $\tau \leq k'$. We apply the local improvement algorithm under this neighborhood in SA every time when the temperature is decreased, and in VNS when the incumbent solution cannot be improved during some iterations. The parameter k' is defined from the interval $[\min(p, n-p), \max(p, n-p)]$.

In order to evaluate the quality of the solutions obtained we rewrite the problem as the mixed integer linear program. We introduce new variables $z_{ij} = p_i x_{ij}$ and include additional constraints with a large positive constant p_{max}:

$$z_{ij} \leq p_{max} x_{ij}, \quad i \in I, j \in J;$$

$$p_{max}(x_{ij} - 1) \leq z_{ij} - p_i \leq p_{max}(1 - x_{ij}), \quad i \in I, j \in J.$$

Now we can use CPLEX software for finding global maximum. Unfortunately, we cannot solve the medium size instances even by supercomputer during 24 hours. Moreover, the best found solutions for this solver were the same or worse than heuristic solutions in all our experiments.

4 Computational experiments

We have tested our method on the randomly generated instances with dimension $n = 100, m = 40, p = 5$. The budget of each client is taken from the interval $[1, 100]$. The transportation costs c_{ij} are generated as Euclidean distances between points i and j on the two-dimensional plane. The points are taken at random from the square 100×100.

Table 1 shows the computational results for 10 benchmarks. The running time is presented for PC Intel Core i7, 2.7 Ghz. As we mentioned above, the branch and bound method (CPLEX) is interrupted after 24 hours. It shows the worse results. The methods SA+VNS and VNS+VNS show the same objective values for all instances. We guess that the global optima are found for all cases. Nevertheless, the VNS+VNS heuristic is faster.

Dimension			SA + VNS	VNS + VNS	CPLEX 12.4
n	m	p	revenue/time	revenue/time	revenue
100	40	5	2245/9h	2245/45min	2226
100	40	5	2259/10h	2259/51min	2259
100	40	5	2019/10h	2019/41min	2019
100	40	5	1533/12h	1533/42min	1508
100	40	5	2386/18h	2386/46min	2313
100	40	5	1960/14h	1960/60min	1949
100	40	5	2179/12h	2179/60min	2142
100	40	5	2139/11h	2139/51min	2139
100	40	5	1895/12h	1895/59min	1877
100	40	5	2209/13h	2209/37min	2209

Table 1. Computational results

References

[1] Aboolean, R., O. Berman, and D. Krass, *Optimizing pricing and locations for competitive service facilities charging uniform price*, J. Oper. Res. Society **59** (2008), 1506–1519.

[2] Eiselt, H. A., G. Laporte, and J.-F. Thisse, *Competitive location models: A framework and bibliography*, Transportation Science, **27** (1993), 44–54.

[3] Fischer, K., *Sequential discrete p-facility models for competitive location planning*, Annals Oper. Res. **111** (2002), 253–270.

[4] Hamjoul, P., P. Hansen, P. Peeters, and J.-F. Thisse, *Uncapacitated plant location under alternative spatial price policies*, Market Sci. **36** (2001), 41–57.

[5] Hansen, P. and N. Mladenovic, *Variable neighborhood search*, European J. Oper. Res. **130** (2001), 449–467.

[6] Hotelling, H. *Stability in competition*, Economic Journal **39** (1929), 41–57.

[7] Kochetov, Yu., E. Alekseeva, T. Levanova, and M. Loresh, *Large neighborhood local search for the p-median problem*, Yugoslav J. Oper. Res. **15** (2005), 53-63.

[8] Lederes, P. J. and J.-F. Thisse, *Competitive location on network under delivered pricing* Oper. Res. Letters. **9** (1990), 147–154.