A Leader-Follower Hub Location Problem Under Fixed Markups

Dimitrije D. Čvokić[1(✉)], Yury A. Kochetov[2,3], and Aleksandr V. Plyasunov[2,3]

[1] University of Banja Luka, Mladena Stojanovića 2, 78000 Banja Luka, Republika Srpska, Bosnia and Herzegovina
dimitriye.chwokitch@yahoo.com
[2] Sobolev Institute of Mathematics, 4, Akad. Koptyug Avenue, 630090 Novosibirsk, Russia
[3] Novosibirsk State University, 2, Pirogova Street, 630090 Novosibirsk, Russia

Abstract. Two competitors, a Leader and a Follower, are sequentially creating their hub and spoke networks to attract customers in a market where prices have fixed markups. Each competitor wants to maximize his profit, rather than a market share. Demand is split according to the logit model. The goal is to find the optimal hub and spoke topology for the Leader. We represent this Stackelberg game as a nonlinear mixed-integer bi-level optimisation problem and show how to reformulate the Follower's problem as a mixed-integer linear program. Exploiting this reformulation, we solve instances based on a synthetic data using the alternating heuristic as a solution approach. Computational results are thoroughly discussed, consequently providing some managerial insights.

Keywords: Hub location · Pricing · Fixed markup · Stackelberg competition · Linear reformulation · Matheuristic

1 Introduction

Competition between firms that use hub and spoke networks has been studied mainly from the sequential location approach. An existing firm, the Leader, serves the demand in some region, and a new one, the Follower, wants to enter. This topic of research is quite fresh, and the first paper on competitive hub location is attributed to Marianov, Serra and ReVelle [1]. Their approach was extended and followed by Eiselt and Marianov in [2], Gelareh, Nickel and Pisinger in [3], and by many others. Sasaki and Fukushima presented a (continuous) Stackelberg Hub Location Problem in [4], in which the incumbent competes with several entrants for profit maximisation. For every route, only one hub was allowed. Adler and Smilowitz introduced in [5] a framework to decide the convenience of merging airlines or creating alliances, using a game-theory-based approach. Later, Sasaki et al. in [6] proposed a problem in which two agents are locating hub-arcs to maximise their respective revenues under the Stackelberg framework, allowing more than one hub on a route.

Y. Kochetov et al. (Eds.): DOOR 2016, LNCS 9869, pp. 350–363, 2016.
DOI: 10.1007/978-3-319-44914-2_28

Here, we consider a sequential hub location and pricing problem in which two competitors, a Leader and a Follower, compete to attract the customers and aim to maximize their profits rather than a market share. The pricing is identified as an important service attribute that affects the client's choice [7–10], as expected. Therefore, we are interested in studying its impact to the optimal hub and spoke topology. In contrast to [11], we assume that the prices are regulated, moreover that markups are fixed.

Regulation is a legal norm intended to shape a conduct that is a by-product of imperfection. It may be used to prescribe or proscribe a conduct, to calibrate incentives, or to change preferences. Common examples of regulation includes control of market entries, *prices*, wages, development, approvals, pollution effects, employment for some people in certain industries, standards of production for some goods, the military forces and devices. For more information we refer the reader to [12–14]. The normative economic theories conclude that the regulations should *encourage competition* where feasible, minimize the cost of information asymmetries, *provide for price structures that improve economic efficiency*, establish regulatory processes that provide for regulation under the law and independence, transparency, *predictability*, legitimacy and credibility of the regulatory system (see [13,15], for example). Price regulation refers to the policy of setting prices by a government agency, legal statute, or regulatory authority. Under such policy, fixed, minimum and maximum prices may be set. Referring to [15,16], a decision may be based on costs, return on investments, or even markups.

As it was previously said, we are interested in a direct price setting as *a form of regulation*, particularly, a scenario where the markups are fixed. Fixing prices is not just a theoretical scenario. When it comes to the transportation industry, a famous example is the IATA (International Air Transport Association) price regulation. That is, several years ago, the price for a non-stop flight from an origin to a destination in a given passenger class was fixed for IATA airlines. The fact that one had or had not to change planes did not affect the price. A passenger could, in principle, use his Lufthansa ticket on a British Airways flight, because tickets were transferable within a fare class, as it was reported by Grammig et al. in [17]. Moreover, Lüer-Villagra and Marianov showed in [11] that if demand is non-elastic and logit model is used for calculating the discrete choice probability, the optimal prices for all routes connecting a particular origin-destination (OD) pair have the same markup.

We note that fixing markups does not mean that prices will be the same, that is they could vary if the routes are composed of several different lines that have different travel costs. As a matter of fact, this approach could be seen as a transition case to a Stackelberg competition in hub location where prices are not regulated. Nevertheless, a hub location or a route opening decision, or even an entrance into a market, can be very dependent on the revenues that a company can obtain using its network. Revenues, in turn, depend on the pricing structure and competitive context, as it was observed in [11].

Following the work of Lüer-Villagra and Marianov in [11], and because this research is still fresh, we will assume that the demand is non-elastic and customers patronize the route by price. Customers' decision process is modelled using a logit model, which is well validated in the transportation literature (see for example [11,18,20]).

Regarding the economies of scale, we use a model in which a constant (flow-independent) discount between hubs and no discount on spokes are considered. In the literature, modeling of the economies of scale in this fashion is addressed as the fundamental approach, which incidentally results in an entirely connected inter-hub network if the objective is the cost minimisation [19]. Most of the researchers use this method of discounting the flow between hubs [19], independent of its magnitude, mainly because of its computational attractiveness and the fact that the search for an entirely successful model is still open [11]. Therefore, we take the same approach in this paper, although we do not expect that hubs have to be completely inter-connected, as we are dealing with the profit maximization problems.

The proposed model is applied to the air passenger industry. However, with slight changes in the discrete choice model, they can be applied to any other industry that benefits from a hub and spoke network structure. We will call this problem a Leader-Follower Hub Location Problem under Fixed Markups (LFHLPuFM).

The contributions of this paper are as follows. Section 2 describes this Stackelberg game. In Sect. 3 we present the mixed-integer linear reformulation of the Follower's problem. After that, in Sect. 4, we describe our solution approach based on the alternating heuristic. In the end, we give some comments and managerial insights.

2 A Leader-Follower Hub Location Problem Under Fixed Markups

The problem is defined over a directed multi-graph $G = G(N, A)$, where N is the non-empty set of nodes and A is the set of arcs that are connecting every pair of nodes in the graph. We assume that for every arc $(i, j) \in A$, there is an opposite arc $(j, i) \in A$. Situations where this does not hold are quite rare and they do not make the problem computationally more attractive. Possible location for hubs are the nodes $i \in N$, and for each of them, there is a fixed cost f_i. The hubs can be shared. We note that the number of hubs to be located is not fixed. Its value is to be determined by the solution of a model. For every arc $(i, j) \in A$ there is a fixed (positive) cost g_{ij} for allocating it as a spoke and a (positive) travel cost per unit of flow c_{ij}. We assume that the travel cost is a non-decreasing function of distance. To model the inter-hub discounts, let $\aleph, \alpha, \psi$ be the discount factors due to flow consolidation in collection (origin to hub), transfer (between hubs), and distribution (hub to destination), respectively. At most two hubs are allowed to be on a single route. The travel cost $c_{ij/kl}$ over a route $i \to k \to l \to j$ is defined as $c_{ij/kl} = \aleph c_{ik} + \alpha c_{kl} + \psi c_{lj}$. It is assumed that pricing is regulated, and a form of regulation is a direct price setting, so that all markups are fixed.

In other words, for every route $i \rightarrow k \rightarrow l \rightarrow j$ there is a fixed markup $\mu_{ij/kl}$. The set of all routes is trimmed to avoid the ones which are impractical, i.e. those routes that have the second arrival point. We define it in a similar fashion as it was done by O'Kelly et al. in [21]

$$I = \{(i,j,k,l) \in N^4 \mid (i = l \wedge l \neq k \wedge k \neq j) \vee (j = k \wedge k \neq l \wedge l \neq i) \vee (i \neq l \wedge k \neq j)\}.$$

On the basis of this set we define the set of valid indices for our routes as $M = \{(i,j,k,l) \in N^4 \mid (i,j,k,l) \in N^4 \backslash I\}$. The sets of valid indices for the possible hubs between the OD pairs $(i,j) \in N^2$ are defined in a similar manner $M_{ij} = \{(k,l) \in N^2 \mid (i,j,k,l) \in N^4 \backslash I\}$. The demand w_{ij} for every OD pair $(i,j) \in N^2$ is assumed to be non-elastic and non-negative. The logit model has a sensitivity parameter Θ that corresponds to the pricing. It has an already known positive value assigned. Higher values of sensitivity parameters mean that the customers are very sensitive to the differences in prices. In other words, they will mostly choose the less expensive routes. Both competitors have a large amount of resources to cover the entire market with their networks. The goal is to maximize the profit, rather than a market share.

This Stackelberg game can be represented as a non-linear mix-integer bi-level mathematical program, where we have that:

- $u_{ij/kl}$ is the fraction of the flow going from $i \in N$ to $j \in N$ through the Leader's hubs located at $k, l \in N$
- $v_{ij/kl}$ is the fraction of the flow going from $i \in N$ to $j \in N$ through the Follower's hubs located at $k, l \in N$
- $x_k = 1$ if the Leader locates a hub at node $k \in N$ and 0 otherwise
- $y_k = 1$ if the Follower locates a hub at node $k \in N$, and 0 otherwise
- $\lambda_{ij} = 1$ if the Leader establishes a direct connection between the nodes $i, j \in N$, where $(i,j) \in A$, and 0 otherwise
- $\zeta_{ij} = 1$ if the Follower establishes a direct connection between the nodes $i, j \in N$, where $(i,j) \in A$, and 0 otherwise

Denote $x = (x_i)_{i \in N},\ y = (y_i)_{i \in N},\ \lambda = (\lambda_{ij})_{i,j \in N},\ \zeta = (\zeta_{ij})_{i,j \in N}$, for short. We propose the following model for the Leader;

$$\max \sum_{(i,j,k,l) \in M} \mu_{ij/kl} w_{ij} u_{ij/kl} - \sum_{i \in N} f_i x_i - \sum_{(i,j) \in A} g_{ij} \lambda_{ij} \tag{1}$$

$$u_{ij/kl} = \frac{x_k x_l \lambda_{ik} \lambda_{kl} \lambda_{lj} e^{-\Theta(c_{ij/kl} + \mu_{ij/kl})}}{\sum\limits_{(s,t) \in M_{ij}} x_s x_t \lambda_{is} \lambda_{st} \lambda_{tj} e^{-\Theta(c_{ij/st} + \mu_{ij/st})} + \gamma^*_{ij}},\ \forall (i,j,k,l) \in M \tag{2}$$

$$\gamma^*_{ij} = \sum_{(k,l) \in M_{ij}} y^*_k y^*_l \zeta^*_{ik} \zeta^*_{kl} \zeta^*_{lj} e^{-\Theta(c_{ij/kl} + \mu_{ij/kl})},\ \forall i,j \in N \tag{3}$$

$$(y^*, \zeta^*) \in F^*(x, \lambda) \tag{4}$$

$$x_i \in \{0,1\}, \quad \forall i \in N \tag{5}$$

$$\lambda_{ij} \in \{0,1\}, \quad \forall (i,j) \in A. \tag{6}$$

Here, y_i^*, ζ_{ij}^* $(i, j \in N)$ are composing the optimal solution for the Follower's problem, for which we propose the subsequent model;

$$\max \sum_{(i,j,k,l)\in M} \mu_{ij/kl} w_{ij} v_{ij/kl} - \sum_{i\in N} f_i y_i - \sum_{(i,j)\in A} g_{ij}\zeta_{ij} \tag{7}$$

$$v_{ij/kl} = \frac{y_k y_l \zeta_{ik}\zeta_{kl}\zeta_{lj} e^{-\Theta(c_{ij/kl}+\mu_{ij/kl})}}{\sum\limits_{(s,t)\in M_{ij}} y_s y_t \zeta_{is}\zeta_{st}\zeta_{tj} e^{-\Theta(c_{ij/st}+\mu_{ij/st})} + \eta_{ij}}, \ \forall (i,j,k,l) \in M \tag{8}$$

$$\eta_{ij} = \sum_{(k,l)\in M_{ij}} x_k x_l \lambda_{ik}\lambda_{kl}\lambda_{lj} e^{-\Theta(c_{ij/kl}+\mu_{ij/kl})}, \ \forall i,j \in N \tag{9}$$

$$y_i \in \{0,1\}, \quad \forall i \in N \tag{10}$$

$$\zeta_{ij} \in \{0,1\}, \quad \forall (i,j) \in A. \tag{11}$$

The objective functions (1) and (7) are representing the profits, which are calculated as a difference between the net income and the network installation costs. Feasible solutions are the tuples $(x, \lambda, y^*, \zeta^*)$ satisfying the constraints (2)–(6). Constraints (2) and (8) are representing the probabilities of choosing the respective routes, according to the logit model. The Eq. (3) represent the impact of the Follower on the Leader's market share. The Leader's impact on the Follower's market share is represented by the Eq. (9). Next, (4) indicates that the Follower chooses the optimal solution for any of the Leader's choice of hubs, where $F^*(x, \lambda)$ represents the set of the Follower's optimal solutions. The rest of the constraint sets are defining the variables' domains.

We note that the Follower's problem may have several optimal solutions, all feasible for a given (x, λ). As a result, the Leader's problem could be ill-posed. Thus, we distinguish two extreme cases:

– cooperative Follower's behaviour (altruistic Follower). In case of multiple optimal solutions, the Follower always selects the one providing the best objective function value for the Leader. We call it the *cooperative optimal solution* to the Follower's problem.
– non-cooperative Follower's behaviour (selfish Follower). In this case, the Follower always selects the solution that provides the worst objective function value for the Leader. We call it the *non-cooperative optimal solution* to the Follower's problem.

One can easily observe that the sum of objective functions in our bi-level program is not a constant. Therefore, the Follower's behaviour should be defined properly, i.e. an auxiliary optimization problem should be defined, as described in [22–24]. The corresponding optimal cooperative solution can be found using a two-stage algorithm.

At Stage 1, for a fixed solution (x, λ), we solve the Follower's problem and calculate the optimal value of its objective function $F(x, \lambda)$.

At Stage 2, for a fixed solution (x, λ), we solve the following *auxiliary problem*

$$\max \sum_{(i,j,k,l)\in M} \mu_{ij/kl} w_{ij} u_{ij/kl} \tag{12}$$

$$u_{ij/kl} = \frac{x_k x_l \lambda_{ik} \lambda_{kl} \lambda_{lj} e^{-\Theta(c_{ij/kl}+\mu_{ij/kl})}}{\sum\limits_{(s,t)\in M_{ij}} y_s y_t \zeta_{is} \zeta_{st} \zeta_{tj} e^{-\Theta(c_{ij/st}+\mu_{ij/st})} + \eta_{ij}}, \ \forall (i,j,k,l) \in M \tag{13}$$

$$\sum_{(i,j,k,l)\in M} \mu_{ij/kl} w_{ij} v_{ij/kl} - \sum_{i\in N} f_i y_i - \sum_{(i,j)\in A} g_{ij} \zeta_{ij} \geq F(x, \lambda) \tag{14}$$

$$v_{ij/kl} = \frac{y_k y_l \zeta_{ik} \zeta_{kl} \zeta_{lj} e^{-\Theta(c_{ij/kl}+\mu_{ij/kl})}}{\sum\limits_{(s,t)\in M_{ij}} y_s y_t \zeta_{is} \zeta_{st} \zeta_{tj} e^{-\Theta(c_{ij/st}+\mu_{ij/st})} + \eta_{ij}}, \ \forall (i,j,k,l) \in M \tag{15}$$

$$\eta_{ij} = \sum_{(k,l)\in M_{ij}} x_s x_t \lambda_{ik} \lambda_{kl} \lambda_{lj} e^{-\Theta(c_{ij/kl}+\mu_{ij/kl})}, \ \forall i,j \in N \tag{16}$$

$$y_i \in \{0,1\}, \quad \forall i \in N \tag{17}$$

$$\zeta_{ij} \in \{0,1\}, \quad \forall (i,j) \in A \tag{18}$$

The corresponding optimal non-cooperative solution can be found using the same two-stage process, except we should solve the minimization problem, instead of the maximization. For a thorough understanding of this topic and the used terminology, we suggest the reader to examine the classic textbook of Dempe [25].

3 Mixed-Integer Linear Reformulation of the Follower's Problem

Suppose that the Leader has made his decision. To estimate his profit (and a market share) we need the Follower's optimal solution. Fortunately, the Follower's problem can be linearised to find the respective optimal solution by a solver.

Introducing a new variable R_{ijkl} (for $(i,j,k,l) \in M$), we can substitute the product $y_k y_l \zeta_{ik} \zeta_{kl} \zeta_{lj}$ in the constraint set (8). This substitution requires the additional sets of constraints

$$R_{ij/kl} - \frac{1}{5}(y_k + y_l + \zeta_{ik} + \zeta_{kl} + \zeta_{lj}) \leq 0, \quad \forall (i,j,k,l) \in M \tag{19}$$

$$R_{ij/kl} - y_k - y_l - \zeta_{ik} - \zeta_{kl} - \zeta_{lj} + 4 \geq 0, \quad \forall (i,j,k,l) \in M \tag{20}$$

$$R_{ij/kl} \in \{0,1\}, \quad \forall (i,j,k,l) \in M \tag{21}$$

where y_i, ζ_{ij} have the same meaning as in (7)–(11).

Now, only the constraints from (8) have non-linear terms. The literature knows several techniques for reformulating the logit-term, which are presented in [26–29]. Recently, Haase and Müller compared those approaches in [20] and their computational study, based on synthetic data, showed that the approach from [27] seems to be promising for solving larger problems. From (8), we directly obtain that the following holds:

$$v_{ij/kl} - \frac{e^{-\Theta(c_{ij/kl}+\mu_{ij/kl})}}{\eta_{ij} + e^{-\Theta(c_{ij/kl}+\mu_{ij/kl})}} R_{ij/kl} \leq 0, \quad \forall(i,j,k,l) \in M, \tag{22}$$

$$v_{ij/kl} \geq 0, \quad \forall(i,j,k,l) \in M. \tag{23}$$

The inequalities (22) are just tighter bounds on v_{ijkl}, than the obvious $v_{ijkl} \leq R_{ijkl}$. Basically, they state that a customer can only choose an established route. The domain of variables v_{ijkl} is specified in (23).

The ratio of the choice probabilities of the two alternatives is independent from other alternatives, i.e. we have that for some $v_{ij/kl}$ and $v_{ij/st}$, for which we know that $R_{ij/st} = 1$, the following identity holds

$$\frac{v_{ij/kl}}{v_{ij/st}} = \frac{e^{-\Theta(c_{ij/kl}+\mu_{ij/kl})}}{e^{-\Theta(c_{ij/st}+\mu_{ij/st})}}. \tag{24}$$

This property is called an Independence of Irrelevant Alternatives (IIA). From the previous equations, we conclude that the following constraints are valid

$$v_{ij/kl} \leq \frac{e^{-\Theta(c_{ij/kl}+\mu_{ij/kl})}}{e^{-\Theta(c_{ij/st}+\mu_{ij/st})}} v_{ij/st} + 1 - R_{ij/st}, \ \forall(k,l),(s,t) \in M_{ij}, \ \forall i,j \in N. \tag{25}$$

These inequalities are valid even if the impractical routes are included because their corresponding values for the choice probabilities v_{ijkl} and establishing the route R_{ijkl} could be both set to zero. It is easy to see that (24) is valid even if we use $u_{ij/kl}$ instead of $v_{ij/kl}$. Thus, we obtain an additional two sets of inequalities that are connecting the choice probabilities of the Follower's routes with the choice probabilities of the Leader's routes and describe the relation between the Leader's routes alone (as in (25) for the Follower). In other words, we have the following constraint sets to be valid for all OD pairs

$$u_{ij/kl} \leq \frac{e^{-\Theta(c_{ij/kl}+\mu_{ij/kl})}}{e^{-\Theta(c_{ij/st}+\mu_{ij/st})}} v_{ij/st} + 1 - R_{ij/st}, \quad \forall(k,l),(s,t) \in M_{ij}, \ \forall i,j \in N \tag{26}$$

$$T_{ij/st} u_{ij/kl} \leq T_{ij/kl} \frac{e^{-\Theta(c_{ij/kl}+\mu_{ij/kl})}}{e^{-\Theta(c_{ij/st}+\mu_{ij/st})}} u_{ij/st}, \quad \forall(k,l),(s,t) \in M_{ij}, \ \forall i,j \in N \tag{27}$$

$$u_{ij/kl} \geq 0, \quad \forall i,j,k,l \in N \tag{28}$$

where $T_{ij/st} = x_s x_t \lambda_{is} \lambda_{st} \lambda_{tj}$, for all $i,j,s,t \in N$. Note that the value of $u_{ij/kl}$ is not known until the Follower makes his move, which is not the case with the product $T_{ij/st}$. Now, for all OD pairs we have that sum of choice probabilities for both competitors is equal to one, that is

$$\sum_{(k,l)\in M_{ij}} u_{ij/kl} + \sum_{(k,l)\in M_{ij}} v_{ij/kl} = 1, \quad \forall i,j \in N. \tag{29}$$

We can introduce a new variable q_{ij} $(i,j \in N)$ to denote the cumulative choice probabilities of the Leader. Furthermore, we can derive new sets of constraints with fewer variables

$$\sum_{(k,l)\in M_{ij}} v_{ij/kl} + q_{ij} \leq 1, \quad \forall i,j \in N \tag{30}$$

$$q_{ij} \geq 0, \quad \forall i,j \in N \tag{31}$$

where (31) defines the new variables' domains. As a matter of fact, (8) can be expressed solely in terms of v_{ijkl} and q_{ij} as a linear constraint

$$v_{ij/kl} - \frac{e^{-\Theta(c_{ij/kl}+\mu_{ij/kl})}}{\eta_{ij}} q_{ij} \leq 0, \quad \forall (i,j,k,l) \in M. \tag{32}$$

One could notice that R_{ijkl} is omitted in the second term from the left-hand side. We do not need that binary variable because we already have that (22) must hold.

Now, taking all this into the account, we proved the following proposition.

Proposition 1. *The Follower's Problem (7)–(11) can be reformulated as a mix-integer linear program with the objective function*

$$\max \sum_{(i,j,k,l)\in M} w_{ij}\mu_{ij/kl}v_{ij/kl} - \sum_{i\in N} f_i y_i - \sum_{(i,j)\in A} g_{ij}\zeta_{ij} \tag{33}$$

subject to (9)–(11), (19)–(23), and (30)–(32).

Although this technique could be used to reformulate the problem of the Leader, there is a question of its usefulness, because the Leader is anticipating the move of the Follower. Nevertheless, it can be easily seen that the same approach will give us the reformulation of the auxiliary problem.

Proposition 2. *The auxiliary problem (12)–(18) can be reformulated as a mix-integer linear program with the objective function*

$$\max \sum_{(i,j,k,l)\in M} \mu_{ij/kl}w_{ij}u_{ij/kl} \tag{34}$$

subject to (14), (16)–(18), (19)–(23), (25)–(29).

The drawback of these reformulations is that they produce a large number of new constraints. The fact that we do not have a constraint on the number of hubs suggest that our models could still be difficult to solve by a solver even for smaller instances.

4 Computational Experiments

The central idea of the matheuristic we used is given in [30,31]. This is an alternating method, where for the solution of the Leader, we compute the best-possible solution for the Follower. Once this has been done, the Leader assumes the role of the Follower and re-optimizes his decision by solving the corresponding problem for the given solution. This process is then repeated until one of the Nash equilibria is discovered or the previously visited solution has been detected. The best what we have found for the Leader is returned as the result of the method. In the beginning, the Leader ignores the Follower.

We conducted the computational experiments to test the method using an artificially generated data. The Cartesian coordinates for the nodes $i \in N$ are randomly generated by a uniform distribution in the interval [0, 100]. The demand is also randomly generated using a (truncated) log-normal distribution on an interval [1, 100], where numbers represent the flow in thousands. The log-normal distribution better corresponds to the real-world data then the uniform distribution when it comes to the passenger flows [32], air traffic demands [33], or airline business [34]. Next, following the work presented in [11,35,36], we took the hub location cost to be the same for all nodes. We could say that the cost of the hub location is proportional to the number of the passengers that will go through the hub. On the other hand, the hub location cost is inversely proportional to the number of hubs (because of competition and load shedding). Therefore, we have a range of cases, where only one hub exists in the market to the case with $|N|$ hubs (a point-to-point network as a trivial hub and spoke topology). Taking that into account, we took the following expression for hub location costs in our experiments $f_i = f = \beta \frac{H_{|N|}}{|N|} \sum_{i,j,k,l \in N} w_{ij}$. The sum represents the total amount of the passengers. In the average, that amount is distributed to the $H_{|N|}/|N|$ hubs, where H_n is the n-th harmonic number. As for $\beta > 0$, it is a coefficient that represents an operating cost per passenger. Considering the running time, we observed in our preliminary investigation that $\beta = 0.06$ happened to be a good choice. Also, we note that this is just a temporary solution for the hub location cost model. The cost of spoke allocation between the pair of nodes i and j was calculated using the expression $\zeta_{ij} = f \frac{c_{ij}/w_{ij}}{\max\limits_{(k,l) \in A} c_{kl}/w_{kl}}$, as in [11,35]. The travel cost is taken to be $c_{ij} = d_{ij}/\max_{i,j \in N} d_{ij}$, where d_{ij} is the Euclidean distance between pair of nodes i and j. This way, normalizing the travel cost, the interval from which we "harvested" the node coordinates becomes irrelevant. In our testing the discount cost values on consolidation and distribution links were $\aleph = \psi = 1$. The experiments were conducted on randomly generated graphs of 5, 6, 7, 8, 9 and 10 nodes. Three values $\alpha \in \{0.1, 0.5, 1.0\}$ are considered for the inter-hub discount factor, and three values, too, for the sensitivity parameter $\Theta \in \{0.25, 1.0, 4.0\}$. For a particular OD pair $(i, j) \in N^2$ all markups were taken to be equal, i.e. $\mu_{ij/kl} = \mu_{ij/st}$ for all $k, l, s, t \in N$. This approach is justified by the results presented in [11]. The markup for a particular OD pair is calculated as percentage of the travel cost of the corresponding non-stop flight. The percentages took values from the set $\{10, 25, 50\}$. For graphs of size 9 and 10 nodes, we used only the smallest and the biggest values of the parameters. In total, we tested 124 different instances. The alternating heuristic was implemented in Python 2.7 using Gurobi 6.5 as the solver, on a 64-bit Windows 8.1 Pro with two 2.00 GHz Six-Core AMD Opteron(tm) processors and 32 GiB of RAM.

It is worth noting that preliminary computational experiments on a model that included the impractical routes showed that they can be a cause for numerical instability, which could lead to wrong solutions and unreasonably long running times.

The Leader's Network Structure. In almost half of the instances tested, the best-reported solution for the Leader was the so-called Entry Deterrence. Slightly less than one-third of the cases had the Nash equilibrium as a solution. We observed from our testing sample that for stronger Leader's positions (lower markups or better developed networks), the harder it was for the Follower to obtain any profit at all. Something quite similar we observed for the running time of the algorithm. For stronger Leader's positions, Gurobi needed more time to find the exact solution and in some cases it even lasted the entire day.

The Role of Inter-hub Economies of Scale. Our computational investigation suggests that the inter-hub economies of scale have a minor impact on the Leader's profit. Unfortunately, we could not observe any solid pattern. It seems that greater values of the price sensitivity parameter combined with bigger markups can boost a little bit the role of the inter-hub economies of scale. For smaller values of markups and sensitivity parameter, the profit becomes more and more locked to one specific value. Also, the results of the computational tests suggest that economies of scale could have an effect to some extent on the Leader's networks solution. In our testing, we did not observe a significant difference in location of hubs for different values of the discount factor, but the resulting hub and spoke topologies were usually less developed for smaller values of the discount factor. We note that sometimes there was no difference at all, or it was the opposite. We could not observe that different values of α had any influence on the cycle length of the alternating heuristics.

The Effect of the Sensitivity Factor. It turns out that the sensitivity to the price differences could have a more significant role when it comes to the profit of the competitors. In most of the cases when the Nash equilibrium occurred as a solution the bigger sensitivity led to the greater profit for the Leader, although not always. The same could be said for the Entry Deterrence type of solution. When it comes to the resulting hub and spoke topologies, in most of the cases hub and spoke networks were the same for different values of Θ, especially for smaller markups. When they were not, the small values of the sensitivity factor usually corresponded to more developed (less translucent) networks. Also, it seems that the chance for the entry deterrence to occur as a solution is greater when Θ takes the smaller values. In contrast to that, the bigger values of Θ corresponded more often to the Nash equilibrium solutions. As in the previous analysis, we can not say that the cycle length is under the influence of the sensitivity factor.

The Role of the Fixed Markups. It is more likely that the Nash equilibrium as a solution will occur when the markups are bigger. For smaller markups, the Entry Deterrence appeared more often as a solution. It was not always the case that the bigger markup led to the greater profit, especially if the solution types were different, but for the same types the bigger markup corresponded to the higher obtained profit. We observed that for the smallest markups considered the alternating heuristic had the minimal number of steps, but for the considerably larger markups we could not observe a regular pattern.

5 Conclusion and Future Research

We present a novel approach to analyse a situation in which two companies compete in a transportation market in a sequential fashion. Here, the goal for the both companies is to maximize their profit by creating the optimal hub and spoke networks. It is assumed that the market is regulated. Because this research is quite fresh, the form of regulation is chosen to be the direct price setting. To be more precise, we assumed that all routes have fixed markups. We have to say that up to our knowledge no one has investigated the effects of regulations to the optimal hub location in a competitive environment. Next, we took that the customers' choice of provider and route depends solely on price and therefore it is possible to predict it by a simple logit model (although including other factors would be very easy). Upon that, we formulated a non-linear mixed integer bi-level program to model this Stackelberg competition. The choice of the solution approach was the alternating heuristic based on the Follower's best response.

The computational investigation showed that discount factor by itself has a relative impact to the solution and a not so sharp-cut role, as it was difficult to observe any regularity. It looks like the sensitivity factor plays a more significant role, but again it is hard to draw any specific conclusions. The markup has a significant effect, as expected. For smaller markups, there is a tendency towards the entry deterrence. Likewise, the bigger markups provided "more space" for the Nash equilibrium to occur. Loosely speaking, if the passengers in the market are less sensitive to the price differences and markups are quite small, than it could be expected that the first-to-enter company will be the only one providing the services. On the other hand, the price sensitive markets with bigger markups set are more "prone" to allow multiple competitors to operate. Colloquially speaking, both of these outcomes could be used as arguments in cooperative games.

From the purely computational point of view, we observed that it might be beneficial to derive a new model that would serve only to find the entry deterrence solution. Long running times confirmed our worries that this problem will not be easily solved for bigger instances by a commercial solver. Therefore, for a more thorough investigation, we have to find a better way to compute the exact solution for the Follower's problem. We intend to put our efforts in finding tighter reformulations of the Follower's model and designing a branch and bound based method that would utilize the structure of the program itself.

Recently, some new interesting results have been obtained by relating the bi-level programs with polynomial and approximation hierarchies [37–42]. The investigation of these relationships is an important area of research. Therefore, we plan to determine the position of our problem in each of them, too. Another direction of the research is oriented towards the other forms of regulation.

Acknowledgements. This research was partially supported by the RFBR grant 16-07-00319.

References

1. Marianov, V., Serra, D., ReVelle, C.: Location of hubs in a competitive environment. Eur. J. Oper. Res. **114**(2), 363–371 (1999). doi:10.1016/S0377-2217(98)00195-7
2. Eiselt, H., Marianov, V.: A conditional p-hub location problem with attraction functions. Comput. Oper. Res. **36**(12), 3128–3135 (2009). doi:10.1016/j.cor.2008.11.014
3. Gelareh, S., Nickel, S., Pisinger, D.: Liner shipping hub network design in a competitive environment. Trans. Res. Part E: Logistics Transp. Rev. **46**(6), 991–1004 (2010). doi:10.1016/j.tre.2010.05.005
4. Sasaki, M., Fukushima, M.: Stackelberg hub location problem. J. Oper. Res. Soc. Japan **44**(4), 390–405 (2001). doi:10.1016/S0453-4514(01)80019-3
5. Adler, N., Smilowitz, K.: Hub-and-spoke network alliances and mergers: price-location competition in the airline industry. Transp. Res. Part B **41**(4), 394–409 (2007). doi:10.1016/j.trb.2006.06.005
6. Sasaki, M., Campbell, J.F., Ernst, A.T., Krishnamoorthy, M.: Hub arc location with competition. Technical report, Nanzan Academic Society (2009)
7. de Menzes, A., Vieira, J.: Willingness to pay for airline services attributes: evidence from a stated preferences choice game. Eur. Transport **39**, 1–13 (2008)
8. Campbell, B., Vigar-Ellis, D.: The importance of choice attributes and the positions of the airlines within the South Africa domestic passenger airline industry as perceived by passengers at Durban International Airport. South. Afr. Bus. Rev. **16**(2), 97–119 (2012)
9. Cullinane, K., Notteboom, T., Sanchez, R., Wilmsmeier, G.: Costs, revenue, service attributes and competition in shipping. Marit. Econ. Logistics **14**, 265–273 (2012). doi:10.1057/mel.2012.7
10. Lambrecht, A., Seim, K., Vilcassim, N., Cheema, A., Chen, Y., Crawford, G., Hosanagar, K., Raghuram, I., Koenigsberg, O., Lee, R., Miravete, E., Sahin, O.: Price discrimination in service industries. Mark Lett. **23**, 423–438 (2012). doi:10.1007/s11002-012-9187-0
11. Lüer-Villagra, A., Marianov, V.: A competitive hub location and pricing problem. Eur. J. Oper. Res. **231**(3), 734–744 (2013). doi:10.1016/j.ejor.2013.06.006
12. Orbach, B.: What is Regulation? 30 Yale J. Regul. Online 1 (2012). Available at SSRN: http://ssrn.com/abstract=2143385
13. Hertog, J.: Review of Economic Theories of Regulation. Discussion Paper, 10–18, Tjalling C. Koopmans Research Institute, Utrecht University School of Economics, Utrecht University, Utrecht (2010)

14. Braithwaite, J., Drahos, P.: Global Business Regulation. Cambridge University Press, Cambridge (2000). ISBN -13: 978-0521784993
15. Khemani, R.S., Shapiro, D.M.: Glossary of Industrial Organisation Economics and Competition Law. OECD Publishing, Paris (1993). ISBN: 9789264137936
16. Treasury, A.: Price regulation of utilities. Econ. Roundup **1**, 57–69 (1999)
17. Grammig, J., Hujer, R., Scheidler, M.: Discrete choice modelling in airline network management. J. Appl. Econ. **20**, 467–486 (2005). doi:10.1002/jae.799
18. Ortúzar, J., Willumsen, L.: Modelling Trasnport, 4th edn. Wiley-Blackwell, West Sussex (2011)
19. Campbell, J., O'Kelly, M.: Twenty-five years of hub location research. Transp. Sci. **24**(2), 153–169 (2012). doi:10.1287/trsc.1120.0410
20. Haase, K., Müller, S.: A comparison of linear reformulations for multinomial logit choice probabilities in facility location models. Eur. J. Oper. Res. **232**, 689–691 (2014). doi:10.1016/j.ejor.2013.08.009
21. O'Kelly, M.E., Bryan, D., Skorin-Kapov, D., Skorin-Kapov, J.: Hub network design with single and multiple allocation: a computational study. Location Sci. **4**(3), 125–138 (1996). doi:10.1016/S0966-8349(96)00015-0
22. Alekseeva, E., Kochetov, Y.: Matheuristics and exact methods for the discrete (r|p)-centroid problem. In: Talbi, E.-G., Brotcorne, L. (eds.) Metaheuristics for bi-level Optimization. SCI, vol. 482, pp. 189–220. Springer, Heidelberg (2013). doi:10.1007/978-3-642-37838-6
23. Beresnev, V., Melnikov, A.: Approximate algorithms for the competitive facility location problem. J. Appl. Ind. Math. **5**(2), 180–190 (2012). doi:10.1134/S1990478911020049
24. Ben-Ayed, O.: Bilevel linear programming. Comput. Oper. Res. **20**(5), 485–501 (1993)
25. Dempe, S.: Foundations of Bilevel Programming. Kluwer Academic, Dordrecht (2002). doi:10.1007/b101970
26. Benati, S., Hansen, P.: The maximum capture problem with random utilities: problem formulation and algorithms. Eur. J. Oper. Res. **143**, 518–530 (2002). doi:10.1016/S0377-2217(01)00340-X
27. Haase, K.: Discrete location planning. Technical report WP-09-09. Institute for Transport and Logistics Studies, University of Sydney (2002)
28. Aros-Vera, F., Marianov, V., Mitchel, J.: p-Hub approach for the optimal park-and-ride facility location problem. Eur. J. Oper. Res. **226**(2), 277–285 (2013). doi:10.1016/j.ejor.2012.11.006
29. Zhang, Y., Berman, O., Verter, V.: The impact of client choice on preventive healthcare facility network design. OR Spectr. **34**, 349–370 (2012). doi:10.1007/s00291-011-0280-1
30. Carrizosa, E., Davydov, I., Kochetov, Y.: A new alternating heuristic for the $(r|p)$-centroid problem on the plane. In: Klatte, D., Lüthi, H.J., Schmedders, K. (eds.) Operations Research Proceedings 2011. Operations Research Proceedings, pp. 275–280 (2012). doi:10.1007/978-3-642-29210-1_44
31. Alekseeva, E., Kochetov, Y., Plyasunov, A.: An exact method for the discrete $(r|p)$-centroid problem. J. Global Optim. **63**(3), 445–460 (2015). doi:10.1007/s10898-013-0130-6
32. Huang, Z., Wu, X., Garcia, A.J., Fik, T.J., Tatern, A.J.: An open-access modeled passenger flow matrix for the global air network in 2010. PLoS ONE **8**(5), e64317 (2013). doi:10.1371/journal.pone.0064317

33. Knudsen, T.: Uncertainities in airport cost analysis. In: Visser, E. (ed.) Transport Decisions in an Age of Uncertainty. Proceedings of the Third World Conference on Transport Research, pp. 362–376. Kluwer Boston, The Hague (1977). doi:10.1007/978-94-009-9707-3
34. Benitez, C.: The Design of a Large Scale Airline Network, Dissertation. Delft University of Technology (2012). doi:10.4233/uuid:c28fef01-c47f-4f92-82ba-6d34d5c4daa4
35. Calik, H., Alumur, S., Kara, B., Karasan, O.: A tabu-search based heuristic for the hub covering problem over incomplete hub networks. Comput. Oper. Res. **36**(12), 3088–3096 (2009). doi:10.1016/j.cor.2008.11.023
36. O'Kelly, M.E.: Hub facility location with fixed costs. Pap. Reg. Sci. **71**(3), 293–306 (1992). doi:10.1007/BF01434269
37. Davydov, I., Kochetov, Y., Plyasunov, A.: On the complexity of the $(r|p)$-centroid problem in the plane. TOP **22**(2), 614–623 (2014). doi:10.1007/s11750-013-0275-y
38. Melnikov, A.: Computational complexity of the discrete competitive facility location problem. J. Appl. Ind. Math. **8**(4), 557–567 (2014). doi:10.1134/S1990478914040139
39. Panin, A.A., Pashchenko, M.G., Plyasunov, A.V.: Bilevel competitive facility location and pricing problems. Autom. Remote Control **75**(4), 715–727 (2014). doi:10.1134/S0005117914040110
40. Caprara, A., Carvalho, M., Lodi, A., Woeginger, G.J.: A study a computational complexity of the bi-level knapsack problem. Siam. J. Optim. **2**, 823–838 (2014). doi:10.1137/130906593
41. Iellamo, S., Alekseeva, E., Chen, L., Coupechoux, M., Kochetov, Y.: Competitive location in cognitive radio networks. 4OR 13(1), 81–110 (2015). doi:10.1007/s10288-014-0268-1
42. Lavlinskii, S.M., Panin, A.A., Plyasunov, A.V.: A bilevel planning model for public-private partnership. Autom. Remote Control **76**(11), 1976–1987 (2015). doi:10.1134/S0005117915110077