Lecture 12.

Vehicle Routing Problems in Supply Chain Management

With the emergence of international markets and the growth of globalization, the management of supply chains has gained increased attention. The high complexity of the underlying ***procurement, production and distribution*** processes, as well as the increasing number of parties involved, create the necessity for efficient decision support systems. Traditionally, these three processes have mostly been solved as single problems with little interactions between them.

The focus on single subproblems neglects structural interdependences and may lead to suboptimal decisions. Nowadays, an efficient and sustainable use of resources is becoming critical for the survival of organizations. This can only be achieved by considering the interdependencies of integrated supply chain services explicitly.

VRP with Time Windows

The VRPTW consists of finding a set of routes for m vehicles such that

- each routes starts and ends at the depot;
- each customer is visited by exactly one vehicle in predefined time window;
- the total demand of customers assigned to a single vehicle does not exceed its loading capacity;
- routes start and end within a driver's working time;
- the sum of the routing cost and of the penalties for time windows violations is minimized.

Decision Variables

- binary flow variables x_{ijk} equal to 1 iff vehicle k traverses from customer i to customer j;
- binary variables y_{ik} equal to 1 iff vehicle k visits customer i and delivers demand q_{ip} for each product $p \in P$;
- time variables $w_{ik} \geq 0$ define the start of service customer i by vehicle $k \in K$;
- penalty variable $u_i \geq 0$ model the delay with respect to the start of service of customer i.

$J = \{0,1, \dots, n, n+1\}$, where 0 and $n+1$ denote the depot,

s_i is service time for customer i.

Mathematical Model

$$\min \sum_{i\in J}\sum_{j\in J} c_{ij} \sum_{k\in K} x_{ijk} + \sum_{i\in J} c_j u_j$$

$$s.t. \qquad \sum_{p\in P}\sum_{j\in J} q_{ip} y_{jk} \leq Q, \quad k\in K;$$

$$\sum_{k\in K} y_{ik} = 1; \quad i\in J\setminus\{0, n+1\}; \qquad \sum_{j\in J} x_{ijk} = y_{jk}; \quad j\in J\setminus\{0\},\ k\in K;$$

$$\sum_{k\in K} y_{ik} = m, \quad i\in\{0, n+1\}; \qquad \sum_{j\in J} x_{ijk} = y_{ik}, \quad j\in\{n+1\},\ k\in K;$$

$$w_{ik} + s_i + t_{ij} \leq w_{jk} + M\left(1 - x_{ijk}\right), \qquad i,j\in J, k\in K;$$

$$a_i \leq \sum_{k\in K} w_{ik} \leq b_i + u_i, \quad i\in J\setminus\{0, n+1\};$$

$$a_i \leq w_{ik} \leq b_i + u_i, \qquad i\in\{0, n+1\}, k\in K;$$

$$x_{ijk}, y_{ik} \in \{0,1\}, \qquad w_{ik}, u_i \geq 0.$$

Lotsizing Problem in Production Planning

In the lotsizing problem we try to optimize the tradeoff between setup and inventory holding costs:

- Producing large batches of products at a time reduces the number of setup operations involved. However, the resulting inventory stock levels and holding costs become higher.
- Producing smaller quantities of products at a time leads to lower average inventory costs, but results in a setup costs increase.

Customers may only be serviced once all required products are ready to be dispatched and shipped. The earlier customer orders are ready, the earlier vehicles may depart and distribute the items to customers.

The Lotsizing Problem and Routing Problem

Let $T = \{1, \dots, |T|\}$ denote the set of time periods per day available for production, and let $T' = T \setminus \{|T|\}$. The total production capacity in t is denote by C_t. Each setup operation cost of product p is denoted by C_p^s and P denotes the set of products.

Decision Variables

$v_{pt} \geq 0$ is the amount of product p produced in time slot t;

$v_{pt}^s \in \{0,1\}$ is equal to 1 iff product p is made in period t;

$h_{pt} \geq 0$ is the stock level of product p at the start of period t;

$o_{pkt} \geq 0$ is the amount of goods of type p to be loaded into vehicle k in time period t;

$z_{kt} \in \{0,1\}$ is equal to 1 iff vehicle k leaves the depot at time t.

Mathematical Model

$$\min \sum_{t\in T}\sum_{p\in P} c_p^s v_{pt}^s + \sum_{i\in J}\sum_{j\in J} c_{ij} \sum_{k\in K} x_{ijk} + \sum_{i\in J} c_i u_i$$

$s.t.$

$$\sum_{p\in P} v_{pt} \le C_t, \quad t\in T;$$

$$v_{pt} \le M v_{pt}^s, \qquad p\in P, t\in T; \qquad h_{p1} = 0, \qquad p\in P;$$

$$h_{pt} + v_{pt} - \sum_{k\in K} o_{pkt} = h_{pt+1}, p\in P, \quad t\in T';$$

$$\sum_{t\in T} z_{kt} \le 1, \qquad k\in K; \qquad w_{0k} = \sum_{t\in T} t z_{kt}, \qquad k\in K;$$

$$o_{pkt} \le \sum_{j\in J} q_{ip} y_{ik} + M(1 - z_{kt}), \; p\in P, k\in K, t\in T;$$

$$o_{pkt} \ge \sum_{i\in J} q_{ip} y_{ik} - M(1 - z_{kt}), \; p\in P, k\in K, t\in T;$$

$o_{pkt}, v_{pt}, h_{pt} \ge 0$; v_{pt}^s, $z_{kt} \in \{0,1\}$; and all constraints of VRPTW

Machine Scheduling Problem in Production Planning

Scheduling is a well-studied problem in operations management. Depending on the characteristics of the underlying production process one can differentiate between ***job shop***, ***flow shop***, and the more general ***open shop*** problems. Scheduling lies at the central core of operational production planning, where jobs need to be assigned to machines, and schedules must fulfill a number of precedence or other technical requirements.

Once a good has been produced, it may be delivered to the next layer of the supply chain, and transportation may only start after *the last task* of the production process is completed. Scheduling decisions interfere with decisions for the routing problem and vice versa.

Machine Scheduling and Routing Problem

The job shop scheduling problem is defined as follows:

P is the set of products which must be processed on machines;

R is the set of machines.

The production process of each individual product p consists of a set of tasks B_p.
Each task $h \in B_p$ must be performed on specific machine $r \in R$;

B^r is the set of tasks for machine $r \in R$;

C_p^r is the unit production cost for product $p \in P$;

d_h is the processing time of task h;

e_p is the first task of product p;

f_p is the last task of product p;

g_p is the earliest starting time of product p.

Tasks must be scheduled on machines in such a way that the sequence of tasks for each product satisfies precedence requirements.

Once the operation of a task has started it cannot be interrupted.

The goal is to determine a feasible schedule, such that each machine may only process one task at a time and the total cost for production and transportation of customer demands is minimized.

Decision variables:

$v_h \geq 0$ denotes the starting time of the operation associated with task h;

$z_{hl} \in \{0,1\}$ equals to 1 iff task h is executed before task l; these variables are defined for all pairs of tasks $h, l \in B^r$ processed on the same machine $r \in R$.

Mathematical Model

$$\min \sum_{p\in P} C_p^r \left(v_{fp} - g_p\right) + \sum_{i\in J}\sum_{j\in J} c_{ij} \sum_{k\in K} x_{ijk} + \sum_{i\in J} c_i u_i$$

$$v_l + d_l \le v_h - M(1 - z_{hl}),\ \ r \in R,\quad h,l \in B^r;$$

$$v_h + d_h \le v_l + M z_{hl},\quad r \in R,\quad h,l \in B^r;$$

$$v_h + d_h \le v_l,\quad h,l \in B_p,\ \ p \in P;$$

$$v_{lp} \ge g_p,\ \ l \in B_p, p \in P;$$

$$w_{0k} \ge v_h + d_h - M(1 - y_{ik}),\qquad k \in K,\ \ i \in J,\ \ p \in P, h \in B_p$$

$$\text{where } h = f_p \ \text{ and } \ q_{ip} > 0;$$

$$v_h \ge 0,\qquad z_{hl} \in \{0,1\},\qquad h,l \in B^r, r \in R;$$

and all constraints of VRPTW.

Hometask*. Design a model for the order batching and routing problem.