Asymptotic properties of the hitting times with taboo for a random walk on an integer lattice

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We consider a symmetric, homogeneous (in space and in time), irreducible random walk $S = \{S(t), t \geq 0\}$ on $\mathbb{Z}^d$, $d \in \mathbb{N}$, having a finite variance of jumps.

Denote by τ the time of the first exit of the process S from the starting point, that is, $\tau := \inf\{t \geq 0 : S(t) \neq S(0)\}$. For $y, z \in \mathbb{Z}^d$, $y \neq z$, we introduce the notion of *hitting time of a point y with a taboo state z* by way of $\tau_{y,z} := \inf\{t \geq \tau : S(t) = y, S(u) \neq z, \tau \leq u \leq t\}$ where as usual $\inf\{t \in \varnothing\} = \infty$. Loosely speaking, for trajectories of the random walk, not passing the taboo state z before the first hitting of the state y, the (extended) random variable $\tau_{y,z}$ equals the time of the point y hitting. For the rest trajectories, one has $\tau_{y,z} = \infty$. Set $H_{x,y,z}(t) := \mathsf{P}(\tau_{y,z} \leq t | S(0) = x)$, $x, y, z \in \mathbb{Z}^d$, $y \neq z$, $t \geq 0$.

Our main results concern the asymptotic properties of the (improper) cumulative distribution functions $H_{x,y,z}(t)$, as $t \to \infty$. Firstly, we find the explicit formula for the limit value $H_{x,y,z}(\infty) = \lim_{t\to\infty} H_{x,y,z}(t)$. Secondly, the asymptotic behavior of $H_{x,y,z}(\infty) - H_{x,y,z}(t)$, as $t \to \infty$, is established for arbitrary $x, y, z \in \mathbb{Z}^d$, $y \neq z$.

In particular, we show that for the random walk on $\mathbb{Z}^d$, except for a simple (nearest-neighbor) random walk on $\mathbb{Z}$, $H_{x,y,z}(\infty)$ belongs to the interval $(0, 1)$. In contrast, for a simple random walk on $\mathbb{Z}$, $H_{x,y,z}(\infty)$ can take values 0, 1 or belong to $(0, 1)$ depending on the relative positions of x, y and z. Moreover, for the random walk on $\mathbb{Z}^d$, except for a simple random walk on $\mathbb{Z}$, the order of decrease of $H_{x,y,z}(\infty) - H_{x,y,z}(t)$, as $t \to \infty$, is determined by dimension d only. However, for a simple random walk on $\mathbb{Z}$, this is specified by the mutual location of x, y and z.

Among the obtained results (see [1]) is the following

Theorem 1. *Let $x, y, z \in \mathbb{Z}^d$ be such that $y \neq z$. Then for the random walk S on $\mathbb{Z}^d$, except for a simple random walk on $\mathbb{Z}$, as $t \to \infty$, one has*

$$H_{x,y,z}(\infty) - H_{x,y,z}(t) \sim \frac{C_1(x-z, y-z)}{\sqrt{t}} \quad \textit{for} \quad d = 1, \tag{1}$$

$$H_{x,y,z}(\infty) - H_{x,y,z}(t) \sim \frac{C_2(x-z, y-z)}{\ln t} \quad \textit{for} \quad d = 2, \tag{2}$$

$$H_{x,y,z}(\infty) - H_{x,y,z}(t) \sim \frac{C_d(x-z, y-z)}{t^{d/2-1}} \quad \textit{for} \quad d \geq 3 \tag{3}$$

where $C_d(\cdot, \cdot)$, $d \in \mathbb{N}$, are some specified positive functions.

Note that establishing these asymptotic properties is pertinent to our recent study of branching random walk on $\mathbb{Z}^d$ with a single source of branching.

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[1] E.Vl. Bulinskaya. *The Hitting Times with Taboo for a Random Walk on an Integer Lattice.* Prépublication de LPMA UPMC no. 1456 (2011); arXiv:1107.1074v1 [math.PR].