Exponential inequalities for the distribution tails of multiple stochastic integrals with Gaussian product-measures

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Let $\xi(t)$ be a centered Gaussian process on $[0,\, 1]$ with the covariance function $\Phi(t,s)$. Denote

$$\Delta\Phi(t,s) = \mathbf{E}\,(\xi(t+\delta) - \xi(t))\,(\xi(s+\delta) - \xi(s)), \quad t, s, t+\delta, s+\delta \in [0,1], \delta > 0.$$

We consider the case when

$$\Delta\Phi(t,s) = \begin{cases} \delta g_1(t) + o(\delta), \ t = s, \\ \delta^2 g_2(t,s) + o(\delta^2), \ \ t \neq s, \end{cases} \tag{1}$$

as $\delta \to 0$, where the functions $g_1(t), g_2(t,s)$ are assumed to be bounded.
We study a multiple stochastic integral of the form

$$I(f) := \int_{[0,\,1]^m} f(t_1, ..., t_m) d\xi(t_1) ... d\xi(t_m)$$

which was defined in [1].

Theorem 1. *Let f be a bounded function and $\Phi(t,s)$ meets (1). Then*

$$\mathbf{P}(|I(f)| > x) \leq C_1(m) \exp\left\{ -\left(\frac{x}{C_2(m, f, g_1, g_2)} \right)^{1/d} \right\}. \tag{2}$$

To prove inequality (2), we use the following version of Chebyshev's power inequality:

$$\mathbf{P}(|I(f_N)| > x) \leq \inf_k \ x^{-2k}\, \mathbf{E}|I(f_N)|^{2k},$$

where $\{f_N\}$ is a sequence of step functions converging to f in certain kernel space. The main problem here is to obtain a suitable upper bound for the even moment on the right-hand side of this inequality.

We also discuss different special cases of the integrating processes $\xi(t)$ when the condition of boundedness of f can be omitted and/or the inequality in (2) can be sharpened.

[1] Borisov, I. S. and Bystrov, A. A.: Constructing a stochastic integral of a non-random function without orthogonality of the noise. *Theory Probab. Appl.*, **50**, 52–80 (2005).