Generalization of Kolmogorov-Prokhorov theorem on the existence of moments of random sums

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Let $\{\xi_n\}_{n=1}^{\infty}$ be the sequence of independent random values (r.v.),

$$S_v = \xi_1 + \xi_2 + \dots + \xi_v,$$

where v is a nonnegative integer-valued r.v. which does not depend on the future under $\{\xi_n\}_{n=1}^{\infty}$. Namely, that for any n, the event $\{v \le n\}$ does not depend on σ-algebra

$$F_{n+1} = \sigma\left(\xi_{n+1}, \xi_{n+2}, \dots\right).$$

The solution of problem of existence of ES_v is given by Kolmogorov-Prokhorov Theorem (see [1], [2]).

In [3], B.A.Sevast'yanov has studied the following problem: let

$$\xi_1, \xi_2, \dots$$

be nonnegative i.i.d.r.v. and $g(x)$ be some function defined on $(0, \infty)$. Under which conditions on the function $g(\cdot)$, the existence of $Eg(\xi_1)$ and of $Eg(v)$ implies $Eg(S_v) < \infty$?

In this talk the following result is given.

Theorem. *Let $g(x)$ be the convex function satisfying the following condition: there exists some constant $C > 0$ such that*

$$g\left(x_1 \cdot x_2\right) \le Cg\left(x_1\right) \cdot g\left(x_2\right) \qquad (*)$$

and v is a nonnegative integer-valued r.v. which does not depend on the future under $\{\xi_n\}_{n=1}^{\infty}$. Then

$$Eg\left(|S_v|\right) \le C \cdot \sum_{n=1}^{\infty} \left(\frac{1}{n} \sum_{k=1}^{n} Eg(\xi_k)\ g(n)\ P(v = n) \right).$$

It is shown by examples that both the condition of convexity and inequality (*) are essential.

[1] *A.N.Kolmogorov, Yu.V.Prokhorov.* On sums of a random number of random terms, Uspekhi Mat. Nauk, (1949) No. 4, 168–172.

[2] *A.A.Borovkov*, *Probability Theory*, Editorial URSS, Moscow, 1999, 697 p.

[3] *B.A.Sevast'yanov*, *Theory of branching processes*, Moscow: Nauka, 1971, 457 p.