On zeros of random functions

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The first part of the talk focuses on different statistical properties of the roots of polynomials $P_n(z) = \sum_0^n \xi_j z^j$ with random iid coefficients ξ_j. It is an old problem going back to A. Bloch and G. Polya's paper of 1932 (On the roots of certain algebraic equations, Proc. London Math. Soc., 33). The recent interest to the problem has been also motivated by different new applications like the theory of quantum chaos (Bogomolny et al, Chaotic dinamics and random polynomials, J. Stat. Phys., 85, 1996).

One of our results (I. Ibragimov, D. Zaporozhets) concerns the asymptotic ($n \to \infty$) distribution of the roots $\{z_{jn}\}$ of P_n in the complex plane. It says that the moduli $\{|z_{jn}|\}$ are concentrated near the unit circle if and only if $\mathbf{E}\ln(1 + |\xi_0|) < \infty$ and that the arguments $\{\arg z_{jn}\}$ are always uniformly distributed on the circle.

The second part of the talk concerns zeros of Gaussian random fields. The main result is the following one (I. Ibragimov, D. Zaporozhets).

Let $G(x)$ be a Gaussian random field defined on a compact set $F \subset \mathbb{R}^d$, $\mathbf{E}G(x) = m(x)$, $\mathbf{Var}G(x) = \sigma^2(x)$. Suppose that with probability 1, $G \in C^1(F)$. Then the mean area $\mathbf{E}\lambda^{d-1}(G^{-1}(0)$ of the set $G^{-1}(0)$ of zeros $G(x)$ satisfies the following relation

$$\mathbf{E}\lambda_{d-1}(G^{-1}(0)) = \frac{1}{\sqrt{2}\sigma}\int_F e^{-\frac{m^2(x)}{2\sigma^2(x)}}\mathbf{E}\big|\nabla\big(\frac{G(x)}{\sigma(x)}\big)\big|\lambda_d dx.$$

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