A construction of stochastic integrals of nonrandom functions in the non-Gaussian case

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We study an approach to define stochastic integrals of the form $\int\limits_a^b f(t)d\xi(t)$, where $f(t)$ is a nonrandom function and $\xi(t)$ is a random process on $[a;\ b]$. We consider random processes of the form

$$\xi(t) = \sum_{k=0}^{\infty} \xi_k \varphi_k(t), \quad t \in [a;\ b].$$

In this representation, the random variables $\{\xi_k\}$ are pairwise orthogonal, with $\mathbb{E}\xi_k^2 = 1$, and every function $\{\varphi_k(t)\}$ has a bounded variation on $[a;\ b]$.

Define the stochastic integral $\int\limits_a^b f(t)d\xi(t)$ by the formula

$$\widetilde{\zeta}(f) := \int\limits_a^b f(t)d\xi(t) \overset{\mathcal{L}_2}{:=} \sum_{k=0}^{\infty} \xi_k \int\limits_a^b f(t)d\varphi_k(t).$$

The integral $\widetilde{\zeta}(f)$ exists if and only if

$$\sum_{k=0}^{\infty} \left(\int\limits_a^b f(t)d\varphi_k(t) \right)^2 < \infty.$$

In [1], it was described another construction of stochastic integral $\zeta(f)$ that includes some known univariate and multiple stochastic-integral models. The main constrain of this scheme is the finiteness of the so-called covariance measure. The integral $\zeta(f)$ was defined for the functions from some functional space S (see [1]). In particular, we proved that, for every continuous function f from S, there exists $\widetilde{\zeta}(f)$ and furthermore, with probability 1,

$$\zeta(f) = \widetilde{\zeta}(f).$$

We demonstrate some examples of random processes $\xi(t)$ such that $\widetilde{\zeta}(f)$ exists but $\zeta(f)$ does not exist.

We also discuss an analogous construction of multiple stochastic integrals.

References

[1] *I. S. Borisov, A. A. Bystrov.* "Constructing a stochastic integral of a nonrandom function without orthogonality of the noise", *Theory Probab. and its Appl.*, 2006, **50**(1), 53-74.