Asymptotic distribution of empirical bridge for regression on order statistics

Artyom Kovalevskii and Evgeny Shatalin

Let $\{\xi_i\}_{i=1}^n$ and $\{\varepsilon_i\}_{i=1}^n$ be independent samples, F be the distribution function of ξ_1, $0 < \mathbf{E}\xi_1^2 < \infty$, $\mathbf{E}\varepsilon_1 = 0$, $0 < \mathbf{Var}\,\varepsilon_1 < \infty$.

Let $\xi_{i:n}$'s be order statistics, θ be an unknown real parameter. We discuss a regression model

$$Y_i = \theta\xi_{i:n} + \varepsilon_i, \quad i = 1, \ldots, n.$$

Let $\widehat{\theta}$ be the least square estimator for θ. Regression residuals are $\widehat{\varepsilon}_i = Y_i - \widehat{\theta}\xi_{i:n}$.

An *empirical bridge* $Z_n^0 = \{Z_n^0(t),\ 0 \le t \le 1\}$ is a polygonal process with verticies

$$\left(\frac{k}{n},\ \frac{\widehat{\Delta}_k - \frac{k}{n}\widehat{\Delta}_n}{s\sqrt{n}}\right),$$

$$\widehat{\Delta}_k = \widehat{\varepsilon}_1 + \ldots + \widehat{\varepsilon}_k,\ s = \sqrt{\overline{\widehat{\varepsilon}^2} - \overline{\widehat{\varepsilon}}^2},\ \widehat{\Delta}_0 = 0, \quad k = 0, \ldots, n.$$

Let

$$GL_F(t) = \int_0^t F^{-1}(s)\,ds, \quad GL_F^0(t) = GL_F(t) - tGL_F(1),$$

$$U_F^0(t) = GL_F^0(t)(\mathbf{E}\xi_1^2)^{-1/2}.$$

We prove weak conwergence in $C(0,1)$ of Z_n^0 to a centered gaussian process $\widehat{Z_F^0}$ with covariance function

$$K_F(t,s) = \min\{t,s\} - ts - U_F^0(s)U_F^0(t),\ s,t \in [0,1].$$