On the sojourn time of a random walk

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Let $\{X_n,\ n \geq 1\}$ be a sequence of independent identically distributed random variables, $S_n = X_1 + \ldots + X_n,\quad n \geq 1$. Given $b \geq 0$, we introduce the sojourn time

$$T_n(b) = \sum_{k=1}^{n} I_{\{S_k > b\}},$$

i.e. the number of points $k,\ 1 \leq k \leq n$, such that $S_k > b$.

The study of the sojourn time (by its type) relates to the boundary problems for random walks. It is well known that, for many boundary problems, the in-depth results can be obtained on using factorization method.

We apply factorization method to the study of the distribution of $T_n(b)$ under the Cramér type condition on the distribution of X_1. We give an explicit representation for the moment generating function of T_n in the case $b = \text{const}$ and study asymptotic properties of the distribution of T_n as $n \to \infty$, $b = b(n) \to \infty$. Both cases $\mathbf{E}X_1 = 0$ and $\mathbf{E}X_1 < 0$ are considered.