Inequalities and large deviation principles for trajectories of processes with independent increments

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Let $S(t)$; $t \geq 0$, be a homogeneous in time process with independent increments. Put

$$\varphi(\lambda) := \mathbf{E}e^{\langle \lambda, S(1) \rangle}, \quad \lambda \in \mathbb{R}^d; \quad \Lambda(\alpha) := \sup_{\lambda}\{\langle \lambda, \alpha \rangle - \ln \varphi(\alpha)\}, \quad \alpha \in \mathbb{R}^d$$

We need Cramér's condition

$[\mathbf{C}_0]$. *For some* $\delta > 0$

$$\varphi(\lambda) < \infty \quad \textit{for all} \quad |\lambda| \leq \delta.$$

As the space of trajectories of random processes we take the space $\mathbb{D}$ of functions $f = f(t);\ 0 \leq t \leq 1$, without discontinuities of the second kind.

As the metric we take $\rho = \rho_F$ introduced in [1] for the space $\mathbb{F}$ which includes $\mathbb{D}$.

Denote $s_T = s_T(t) := \frac{1}{T} S(Tt)$, $0 \leq t \leq 1$.

The main result of the talk is as follows:

Under condition $[\mathbf{C}_0]$ *the family* $\{s_T\}_{T \geq 1}$ *satisfies large deviation principles (local and extended (see [2])) in the metric space* $(\mathbb{D}, \rho)$ *with parameters* $(T,\ J)$*, where* $J = J^{\Lambda}(f)$ *is the deviation integral introduced in [3].*

References

[1] *Borovkov A.A.*, "The convergence of distributions of functionals on stochastic processis," Russian Math. Surveys, **27**, N 1, pp. 1-42 (1972).

[2] *Borovkov A.A., Mogulskii A.A.*, "On large deviations principles in metric spaces," Siberian Mathematical Journal, Vol. 51, N 6, pp. 989—1003, (2010).

[3] *Borovkov A.A., Mogulskii A.A.*, "Properties of a functional of trajectories which arises in studying the probabilities of large deviations of random walks," Siberian Mathematical Journal, Vol. 52, N 4, pp., (2011).