# BIT FLIPPING MODELS

Anton Muratov, Sergey Zuyev[1]

In many areas of engineering and science one faces with an array of devices which possess a few states. In the simplest case these could be on-off or idle-activated states, in other situations broken or 'dead' states are added. If the activation-deactivation or breakage cycles produce in a random fashion, a natural question to ask is when, if at all, the system of devices, which we call *bits*, recovers to some initial or *ground state*. By this we usually mean the state when all the bits are not active, allowing only for idling and/or broken bits to be seen. The time to recover may assume infinite values when the system actually does not recover or finite values. In the former case we speak of *transient* behaviour of the system. In the latter case, depending of whether the mean of the recover time exists or not, we speak of *positive* or *null-recurrence* of the system. The terminology is borrowed from Markov chains setting and the above classification is tightly related to the exact random mechanism governing the change of bits' states.

To be more specific, we consider two basic models. In both models we deal with a countably infinite array of bits which we index by the natural numbers $\mathbb{N} = \{1, 2, \dots\}$. Initially, the system is in the ground state: all the bits are idling. At each step the number of a bit to change its state is a random variable with a distribution $\mathcal{P} = (p_1, p_2, \dots)$ on $\mathbb{N} = \{1, 2, \dots\}$. We assume that the bits are numbered in such a way that $p_1 \geq p_2 \geq \dots$. In the first model, Bit Flipping (BF), bits only alternate between two states, *idle* and *active*. In the second, Damaged Bits (DB), we let a bit become *dead* after being selected for a second time, i.e. the dynamics is idle→active→dead. We assume that the support of $\mathcal{P}$ is unbounded, otherwise our models are described by a finite state Markov chain with evident behaviour. The main quantity of interest the number of steps $\tau$ to return to the ground state in the *recurrent* case, or the rate of growth of the number of "active" bits in the *transient* case.

Main result for both models is the existence of the *critical decay* of $p_k$ at which the model changes from transient to the recurrent behaviour. In BF the critical decay is $p_k \sim C2^{-k}$. For DB it is given by $p_k \sim C \exp(-\frac{x}{\psi(x)})$ for some slowly varying $\psi(x)$. Besides, the conditions of a central limit theorem are fulfilled for the transient behaviour.

[1]Anton Muratov, Sergey Zuyev, *Chalmers University of Technology, MV, 412-96 Gothenburg, Sweden*