STRUCTURE OF THE GIBBS MEASURE IN THE SHERRINGTON-KIRKPATRICK MODEL

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In the Sherrington-Kirkpatrick spin glass model, given i.i.d. standard normal r.v.s $(g_{i,j})$, one considers a Gaussian process

$$H_N(\vec{\sigma}) = \sum_{i,j=1}^{N} g_{i,j}\sigma_i\sigma_j$$

indexed by vectors $\vec{\sigma} = (\sigma_1, \ldots, \sigma_N) \in \{-1, +1\}^N/\sqrt{N}$ of length one and defines a random probability measure, called the Gibbs measure,

$$G_N(\{\vec{\sigma}\}) = \frac{\exp(\beta\sqrt{N}H_N(\vec{\sigma}))}{Z_N}$$

for some inverse temperature parameter $\beta > 0$. As the size of the system N goes to infinity, one would like to understand the geometry of the set on which the measure G_N concentrates or, in one possible interpretation, given an i.i.d. sample $(\vec{\sigma}^l)$ from G_N, one would like to find the asymptotic distribution of the Gram matrix $(\vec{\sigma}^l \cdot \vec{\sigma}^{l'})_{l,l' \geq 1}$ under all the randomness involved, $\mathbb{E}G_N^{\otimes\infty}$. This distribution was predicted by the Italian physicist Giorgio Parisi and the main feature of his theory is that the measure G_N in the limit must concentrate on an ultrametric set. I will review several results from recent years that partially confirm Parisi's predictions.