Transient phenomena for random walks in the absence of the expected value of jumps

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Let $\xi, \xi_1, \xi_2, \dots$ be independent identically distributed random variables, and

$$S_n := \sum_{j=1}^{n} \xi_j, \qquad \overline{S} := \sup_{n \geq 0} S_n.$$

If $\mathbf{E}\xi = -a < 0$ then we call transient those phenomena that happen to the distribution $\overline{S}$ as $a \to 0$ and $\overline{S}$ tends to infinity in probability. We consider the case when $\mathbf{E}\xi$ fails to exist and study transient phenomena as $a \to 0$ for the following two random walk models:

1. The first model assumes that ξ_j can be represented as $\xi_j = \zeta_j + a\eta_j$, where $\zeta_1, \zeta_2, \dots$ and $\eta_1, \eta_2, \dots$ are two independent sequences of independent random variables, identically distributed in each sequence, such that $\sup_{n\geq 0} \sum_{j=1}^{n} \zeta_j = \infty$, $\sup_{n\geq 0} \sum_{j=1}^{n} \eta_j = \infty$, and $\overline{S} < \infty$ almost surely.
2. In the second model we consider a triangular array scheme with parameter a and assume that the right tail distribution $\mathbf{P}(\xi_j \geq t) \sim V(t)$ as $t \to \infty$ depends weakly on a, while the left tail distribution is $\mathbf{P}(\xi_j < -t) = W(t/a)$, where V and W are regularly varying functions and $\overline{S} < \infty$ almost surely for every fixed $a > 0$. We obtain some results for identically and differently distributed ξ_j.

[1] A. A. Borovkov, P. S. Ruzankin. Transient phenomena for random walks in the absence of the expected value of jumps // Siberian Mathematical Journal, 2009, 50:5, 776-797.