ABOUT RATES OF CONVERGENCE IN INVARIANCE PRINCIPLES

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Many of publications devoted to estimates in the invariance principle are inspired by the works of Yu.V.Prokhorov (1956), A.V.Skorohod (1961) and A.A.Borovkov (1973). Several asymptotically unimprovable estimates in i.i.d. case were obtained by J.Komlos, P.Major and G.Tusnady (1975-76).

The first aim of this work is to present an estimate which alone easily imply all mentioned results of KMT. We need the following assumptions

a) *$H(\cdot)$ is an even nonnegative function and $H(x_0) > 0$ for some $x_0 > 0$;*

b) *$\alpha > 2$ and the function $h_\alpha(x) = H(x)/x^\alpha$ is nondecreasing for $x \geq x_0$;*

c) *the function $h(x) = x^{-1} \log H(x)$ is nonincreasing for $x \geq x_0$;*

d) *$X, X_1, X_2, \ldots$ are independent random variables with common distribution function F such that $\mathbf{E}X = 0$, $\mathbf{E}X^2 = 1$ and $\mathbf{E}H(X) < \infty$;*

e) *$\mathbf{E}\big[H(X);\, |X| \geq y_0\big] \leq h_\alpha^{2/\alpha}(y_0)/3$ for some $y_0 \geq x_0$;*

f) *$y_n \geq y_0$ and $n\mathbf{E}\big[H(X);\, |X| \geq y_n\big] \leq H(y_n)$ for all $n \geq 1$;*

g) *the random process $S(\cdot)$ is monotone on each interval $[n, n{+}1]$ with $S(0) = 0$ and $S(n) = X_1 + \cdots + X_n$ for all $n \geq 1$.*

Theorem. *Suppose that conditions* a) – g) *are fulfilled. Then for every n it is possible to construct a Wiener process $W_{n,y_n,H,F}(\cdot)$ such that*

$$\forall x \geq y_n \quad \forall b \geq 1 \quad \mathbf{P}\Big[\sup_{t \leq n} \big|S(t) - W_{n,y_n,H,F}(t)\big| > C_{abs} bx\Big]$$
$$\leq n\mathbf{P}[|X| > x] + \frac{n}{x^2 H^{bc(\alpha)}(x)} + \frac{n}{x^2 e^{bx/y_0}},$$

where $c(\alpha) = (\alpha - 2)/\alpha$ and $C_{abs} < \infty$ is an absolute constant.

In particular, subdividing the summands into blocks we obtain

Corollary. *Assume that conditions* a) – g) *hold while numbers $\{v_j\}$ satisfy*

$$\forall j \geq 1 \;\; v_{j+1} \geq v_j \geq y_j, \quad \sum\nolimits_{j \geq 1} \mathbf{P}[|X| > v_j] < \infty, \quad \sum\nolimits_{j \geq 1} \frac{1}{H^r(v_j)} < \infty$$

for some $r \geq c(\alpha)$. Then there exists a Wiener process $W_{v_\bullet,H,F}(\cdot)$, such that

$$\limsup_{n \to \infty} \frac{\big|S(n) - W_{v_\bullet,H,F}(n)\big|}{r v_n / c(\alpha) + y_0 \log n} \leq 4C_{abs} < \infty \qquad \text{a.s.}$$

The main goal of the work is to obtain more general results for sums of non-identically distributed random variables.

All proofs are based on the key estimates of A.I.Sakhanenko (1983–89).