Asymptotic properties of ladder epochs of random walks with small negative drift

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We shall consider a family of random walks $\{S^{(a)}, a \in [0, a_0]\}$ with drift $-a$, that is, $\mathbf{E}S_1^{(a)} = -a$, and investigate the asymptotic behaviour, as $a \to 0$, of the probability $\mathbf{P}(\tau^{(a)} > n)$ for $n = n(a)$, where $\tau^{(a)}$ is the first descending ladder epoch of $S^{(a)}$, i.e.,

$$\tau^{(a)} = \min\{k \geq 1 : S_k^{(a)} < 0\}.$$

Assuming that $S_n^{(a)} = S_n - na$ and S_n/c_n converges to a stable law, we identify three regimes:

- If $\frac{an}{c_n} \to 0$, then $\mathbf{P}(\tau^{(a)} > n) \sim \mathbf{P}(\tau^{(a)} > n)$.
- If $\frac{an}{c_n} \to u \in (0, \infty)$, then $\mathbf{P}(\tau^{(a)} > n) \sim G(u)\mathbf{P}(\tau^{(a)} > n)$.
- If $\frac{an}{c_n} \to \infty$, then, under some addtional conditions, $\mathbf{P}(\tau^{(a)} > n) \sim \mathbf{E}\tau^{(a)}\mathbf{P}(X > an)$.

As a consequence, we also obtain the growth rates of the moments of $\tau^{(a)}$.