Absorption in random nonlinear medium

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This communication is dedicated to a nonlinear boundary problem describing diffusion or heat transfer in a random absorbing medium: for $x \in G$ and $t > 0$

$$\partial_t \left(|u_\varepsilon|^{\gamma-2} u_\varepsilon \right) = \operatorname{div} \left(A_\varepsilon (x, u_\varepsilon) |\nabla u_\varepsilon|^{p-2} \nabla u_\varepsilon \right) - B_\varepsilon^\sigma |u_\varepsilon|^{\sigma-2} u_\varepsilon,$$
$$u_\varepsilon|_{t=0} = u_0 \in L^{\gamma+k}(G), k \in \mathbb{R}_+, \ u_\varepsilon|_{\partial G} = 0,$$

where $G \Subset \mathbb{R}^d$ is a bounded domain with regular boundary. The solution belongs to a Sobolev space with variable exponents [1]. The random field $B_\varepsilon \geq 0$ characterizes distribution of the absorbing agent, and the small parameter $\varepsilon > 0$ governs its dispersity. The variable exponents $\sigma(x) < \gamma(x) < p(x)$ are separated by positive gaps and Hölder continuous.

For constant exponents, a solution vanishes totally after a finite time if $S > 0$ is separated from zero [2]. Finite extinction time (and other localization effects) survive the passage to Sobolev spaces with variable exponents [3].

The effect is lost if absorption is weak on a massive set. Nevertheless, the solution can "imitate" extinction in finite time if B_ε is separated from zero on a disperse fine-grained set even if it vanishes on its complement: it passes through a phase of rapid decay, remaining very small when it ends. For constant exponents of nonlinearity, this was proved in [4]; here this approach is extended to variable exponents. Techniques combine those of Alt and Luckhaus with results of [1, 3].

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