The functional limit theorem for canonical U-processes of dependent observations

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The talk is devoted to proving the invariance principle for *a sequence of normalized U-statistics* (this is the so-called *U-process* on $[0,1]$)

$$U_n(t) := n^{-m/2} \sum_{1 \le i_1 \ne} \dots \sum_{\ne i_m \le [nt]} f(X_{i_1}, \dots, X_{i_m}), \qquad t \in [0,1],$$

of an arbitrary order $m \ge 2$ with canonical (degenerate) kernels f, defined on samples from a stationary sequence of observations $\{X_i\}_{i=1}^{\infty}$ satisfying φ- and α-mixing.

The corresponding limit distribution coincides with that of polynomial

$$U(t) := \sum_{k_1=1}^{\infty} \dots \sum_{k_m=1}^{\infty} f_{k_1 \dots k_m} t^{m/2} \Pi_{j=1}^{\infty} H_{v_j(i_1,\dots,i_m)}(t^{-1/2} w_j(t)), \qquad t \in [0,1],$$

of a sequence of dependent standard Wiener processes $\{w_i(t)\}_{i=1}^{\infty}$ with a known joint covariance, where $\{f_{k_1 \dots k_m}\}$ are the expansion coefficients of the kernel f with respect to an orthonormal basis in $L_2(\mathcal{L}(X_1)^m)$, $H_k(x)$ are Hermite polynomials, and $v_j(i_1, \dots, i_m)$ is the number of subscripts among $i_1, \dots, i_m$ equal to j.

It means that, for any measurable in $D[0,1]$ functional $g(\cdot)$, continuous at the points of $C[0,1]$, the sequence $g(U_n)$ converges in distribution to the random variable $g(U)$ and the corresponding multiple series a.s. converges for every $t \in [0,1]$, and moreover, the process $U(t)$ is a.s. continuous.

In the 1980's, there were obtained some limit theorems for U-statistics of arbitrary orders with canonical kernels and independent observations. The limit random variables are represented as infinite polynomials of independent Wiener processes (A.F. Ronzhin, 1986) or multiple stochastic integrals w.r.t. the stochastic product-measure generated by the so-called two-parametric Kiefer process (M. Denker, 1985). There is also a number of results on the limiting behavior of canonical U-statistics for independent observations (see H. Rubin and R. Vitale, 1980) and for φ-, α-mixing trials (I.S. Borisov and N.V. Volodko, 2008).