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List of Participants

DETECTION THRESHOLDS IN MATRIX COMPLETION

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This is a joint work with Simon Coste and Raj Rao Nadakuditi. Let X be a rectangular matrix of size $n \times m$ and Y be the random matrix where each entry of X is multiplied by an independent $0-1$ Bernoulli random variable with parameter $1/2$. In many practical settings, the spectrum of the matrix $Y(X-Y)^*$ conveys more relevant information on the structure of X than the spectrum of XX^* used in principal component analysis. We illustrate this striking phenomenon on the matrix completion problem where the matrix X is equal to a matrix T on a random subset of entries of size dn and all other entries of X are equal to zero. In the regime where the ratio n/m is of order 1 and provided that a usual incoherence assumption holds for the matrix T, we show that the eigenvalues of $Y(X-Y)^*$ with modulus greater than an explicit threshold are asymptotically equal to the eigenvalues of TT^* greater than this threshold, and the associated eigenvectors are aligned. It notably holds in a very sparse regime where d is of order 1. This breaks the theoretical-information limit d of order $\log n$ for recovery well-known in the literature. We also define an improved version of this asymmetric principal component analysis which allows to remove the Bernoulli random variables and improve by a constant factor the detection threshold at the cost of increasing the dimension of the asymmetric matrix.

GAUSSIAN PROCESS APPROXIMATIONS FOR MULTICOLOR PÓLYA URN MODELS

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Motivated by mathematical tissue growth modelling, we consider the problem of approximating the dynamics of multicolor Pólya urn processes that start with large numbers of balls of different colors and run for a long time. Using strong approximation theorems for empirical and quantile processes, we establish Gaussian process approximations for the Pólya urn processes. The approximating processes are sums of a multivariate Brownian motion process and an independent linear drift with a random Gaussian coefficient. The dominating term between the two depends on the ratio of the number of time steps n to the initial number of balls N in the urn. We also establish an upper bound of the form $c(n^{-1/2} + N^{-1/2})$ for the maximum deviation over the class of convex Borel sets of the step n urn composition distribution from the approximating normal law. [Preprint available as arXiv:1912.09665]

MAXIMAL DISPLACEMENT OF A CATALYTIC BRANCHING RANDOM WALK

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The talk is devoted to evolution of a particle population range. This problem goes back to the classical paper by A. N. Kolmogorov, I. G. Petrovski and N. S. Piskunov (1937). Important models were proposed and studied by K. B. Athreya, D. Bertacchi, J. D. Biggins, S. Foss, B. Mallein, S. A. Molchanov, Z. Shi, V. A. Topchij, V. A. Vatutin, E. B. Yarovaya, F. Zucca and other researchers. We provide a survey of the previous works and then concentrate on our new results. A catalytic branching random walk (CBRW) on $\mathbb{Z}^d$, $d \in \mathbb{N}$, is exploited as a mathematical model of the phenomena. The distinctive model trait is a finite number of catalysts present at fixed lattice points such that a particle in CBRW may produce offspring or die just at the location of a catalyst. Outside the catalysts a particle performs a random walk on $\mathbb{Z}^d$ until hitting a catalyst.

A CBRW can be classified as supercritical, critical or subcritical (see [1]). In a supercritical CBRW only, the particle population survives globally and locally with positive probability, whereas in subcritical and critical CBRWs the population degenerates locally with probability one although it can survive globally with positive probability. Whenever in a supercritical CBRW the population survives, it increases exponentially-fast in time. In this case the study of area settling in time by the particle population is reduced to analysis of the rate of the population spread. As shown in [2] and [3], it depends on "heaviness" of the distribution tails of the random walk. Correspondingly, the growth of the particle population "front" covers the scale from linear to exponential in time mode.

In critical and subcritical CBRWs the problem of inhabited territory variation is posed in another way. The main interest focuses on the maximal displacement of the particles in the CBRW during the whole history of particles existence (see [4]). We reveal new effects in the asymptotic behavior of the distribution tail of the maximal displacement which are not observed in the model of a non-catalytic branching random walk.

Methods of investigation include renewal theory, "many-to-one" lemma, analysis of solution to a system of integral equations, Laplace transform, convex analysis, martingale change of measure, large deviation theory and the coupling method.

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SOME APPROXIMATION RESULTS FOR SUBCRITICAL ERDÖS-RÉNYI RANDOM GRAPHS VIA STEIN'S METHOD

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We consider two applications of Stein's method for probability approximation to statistics of interest in a subcritical Erdös-Rényi random graph model. Our first application is the approximation of the length of the shortest path between two uniformly chosen vertices (conditional on these being in the same component of the graph) by a geometric distribution. Our second application is to the approximation of the size of the component containing a uniformly chosen vertex by a Borel distribution. This represents the first development of Stein's method for Borel approximation. Based on joint work with Seva Shneer.

NON-STANDARD LIMITS FOR A FAMILY OF AUTOREGRESSIVE STOCHASTIC SEQUENCES

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We consider a family of multivariate autoregressive stochastic sequences that restart when hit a neighbourhood of the origin, and study their distributional limits when the autoregressive coefficient tends to one, the noise scaling parameter tends to zero, and the neighbourhood size varies. We obtain a non-standard limit theorem where the limiting distribution is a mixture of an atomic distribution and an absolutely continuous distribution whose marginals, in turn, are mixtures of distributions of signed absolute values of normal random variables. In particular, we provide conditions for the limiting distribution to be normal, like in the case where there is no the restart mechanism. The main theorem is accompanied by a number of examples and auxiliary results of their own interest.

This is a joint work with Matthias Schulte (Heriot-Watt University).

LONGEST AND HEAVIEST PATHS IN RANDOM DIRECTED GRAPHS

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In this talk we give an overview of research in the area of random directed graphs with possibly random edge weights. We are interested in longest paths between two vertices (or heaviest paths if there are weights). Typically, the longest path satisfies a law of large numbers and a central limit theorem (which gives a non-normal distribution in the limit if the vertex set is not one-dimensional). The constant in the law of large numbers as a function of the graph parameters and weight distributions cannot be computed explicitly except, perhaps, in very simple cases. A lot of work has been done in obtaining bounds and in studying its behaviour. For example, is it a smooth function of the connectivity parameter p? These kind of graphs appear in several areas: in computer science, in statistical physics, in performance evaluation of computer systems and in mathematical ecology. They originated in a paper by Barak and Erdos but have also been studied independently, in connection with the applications above. They questions asked are related to the so-called last passage percolation problems because we can interpret “longest” in a time sense (what’s the worst case road that will take us from a point to another point?). As such, it is not surprising that in some cases, the limiting behaviour is related to limits of large random matrices. However, the complete picture is not understood and so open problems will also be presented.

SOME INEQUALITIES IN BOUNDARY CROSSING PROBLEMS FOR RANDOM WALKS

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We discuss the accuracy of an upper bound [1] for the tail distribution of trajectory supremum of random walk with negative drift, and then give two-sided inequalities for the probability to leave the strip through its upper boundary. In addition, we present new two-sided inequalities for the average sample number of the sequential probability ratio test [2].

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A CONTINUOUS-TIME VERSION OF THE DERRIDA – RETAUX MODEL

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The Derrida–Retaux model is a simple model of renormalisation on a tree, introduced in statistical physics to study a phase transition in a polymer model. Multiple conjectures on this family of models remain open. We introduce a continuous-time version of this model, which happen to be exactly solvable. We will see some of the results that can be obtained in this solvable model, with a focus on the behavior at criticality of the process.

POINTS AND LINES CONFIGURATIONS FOR PERPENDICULAR BISECTORS OF CONVEX CYCLIC POLYGONS

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We take interest in the topological configurations of points and lines that may arise when placing n points on a circle and drawing the n perpendicular bisectors of the sides of the corresponding convex cyclic n-gon. On a deterministic level, we characterize all such configurations. On a probabilistic level, we consider random configurations obtained either by sampling the points uniformly at random on the circle or by sampling a realizable configuration uniformly at random. We provide exact and asymptotic formulas describing these random configurations.

This is joint work with Paul Melotti (Université de Fribourg) and Paul Thévenin (École Polytechnique), based on the preprint arxiv:2003.11006.

NEW ASYMPTOTICS OF FIRST-PASSAGE TIMES FOR RANDOM WALKS IN THE TRIANGULAR ARRAY SETTING

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Denote by $T(n)$ the first-passage time over n-th moving boundary for n-th random walk constructed by consecutive sums in n-th row of a standard triangular array of independent random variables which satisfies the classical Lindeberg condition. We investigate the asymptotic behaviour of the tail probability that $T(n) > n$. Remind, that in the important case when the random walk is constructed by a fixed sequence of independent random variables the universal asymptotic for the investigated tail probability was found in the joint work of Denisov, Sakhanenko and Wachtel in Ann. Probab., 2018. Now we present conditions under which similar asymptotic holds in the triangular array setting. On the other hand, we show that different asymptotics for the investigated tail probabilities may also take place in this more general setting.

ON THE MODIFIED PALM VERSION

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The Palm version of a stationary random measure is an important tool in probability. It is however not well known that there are in fact two Palm versions, with related but different interpretations. For lack of better terms, call the well known version standard and the less known version modified.

In this talk we shall first compare the two Palm versions and their interpretations, using coin tosses as a transparent example, and then focus on the modified Palm version. The concepts of shift-coupling and mass-stationarity will play a key role.

BRANCHING PROCESSES IN RANDOM ENVIRONMENT WITH IMMIGRATION: LIFE-PERIODS AND SURVIVAL OF A SINGLE FAMILY

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A branching process with immigration which evolve in an i.i.d. random environment is considered. Assuming that immigration is not allowed when there are no individuals in the aboriginal population we investigate the tail distribution of the so-called life period of the process, i.e., the length of the time interval between the moment when the process is initiated by a positive number of particles and the moment when there are no individuals in the population for the first time.

We also consider a branching process in an i.i.d. random environment, in which one immigrant arrives at each generation and are interested in the event $\mathcal{A}_i(n)$ that all individuals alive at time n are descendants of the immigrant which joined the population at time $i < n$. We study the asymptotic probability of this event when n is large and i follows different asymptotics which may be related to n (i fixed, close to n, or going to infinity but far from n).

LIMIT THEOREMS FOR L-PROCESSES

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We consider L-processes including, as special cases, generalized Lorenz curves as well as classical L-statistics (linear combinations of functions of order statistics). These processes are based on weakly dependent random variables.

Let $X_1, X_2, \ldots$ be a sequence of random variables with common distribution function F and let $X_{n:1} \leq \ldots \leq X_{n:n}$ be the order statistics based on the sample $\{X_i; i \leq n\}$. We consider a random function on $[0,1]$ called the L -process and defined explicitly by the formula:

$$L_{n,h}(t) = \frac{1}{n} \sum_{i=1}^{n} c_{n,i}(t) h\left(X_{n:i}\right), \quad 0 \leq t \leq 1,$$

with the coefficients

$$c_{n,i}(t) = n \int_{(i-1)/n}^{i/n} J(s,t) ds.$$

L-processes $L_{n,h}(t)$ were considered in [1] as a generalization of a wide class of statistics.

When t is fixed, or when the kernel $J(s,t)$ does not depend on t, then $L_{n,h}(t)$ is a classical L -statistic:

$$L_{n,h}(t) \equiv L_n = \frac{1}{n} \sum_{i=1}^{n} c_{n,i} h\left(X_{n:i}\right).$$

Strong laws for L-statistics based on the ergodic stationary and φ -mixing sequences were obtained in [2].

Note also that if $J(s,t) = I(s \leq t)$ and $h(t) = t$, then $L_{n,h}(t)$ is the (empirical) generalized Lorenz curve (see [3,5]) :

$$L_{n,h}(t) \equiv L_n(t) = \int_0^t F_n^{-1}(s) ds, \quad t \in [0,1],$$

where F_n^{-1} denotes the empirical quantile function corresponding to the empirical distribution function F_n based on a sample $\{X_i; i \leq n\}$.

Strong uniform convergence of $L_n(t)$ to the theoretical generalized Lorenz curve was proved for i.i.d. case by Goldie (see [4]). Davydov and Zitikis [3] proved this strong convergence under the assumption that the sequence $\{X_n\}_{n\geq 1}$ is strictly stationary and ergodic.

The aim of this talk is to prove strong laws for L-processes based on φ-mixing and α-mixing sequences.

In our main result we give sufficient conditions for the following convergence:

$$\sup_{0\leq t\leq 1} |L_{n,h}(t) - L_h(t)| \to 0 \quad \text{a.s.}$$

where $L_h(t) = \int_0^1 J(s,t)h(F^{-1}(s))ds,\ t \in [0,1]$. This result immediately implies the SLLN for L-statistics and the uniform convergence of $L_n(t)$.

We also obtain Glivenko – Cantelli-type theorem for φ- and α-mixing sequences:

Let $\{X_n\}_{n\geq 1}$ be a φ- or α-mixing sequence of identically distributed random variables and let $\varphi(n) \to 0$ or $\sum_{n=1}^{\infty} n^{-1}\alpha_n < \infty$, respectively. Then

$$\sup_{-\infty<x<\infty} |F_n(x) - F(x)| \to 0 \quad \text{a.s}$$

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ASYMPTOTIC ANALYSIS OF THE DISTRIBUTION OF THE SOJOURN TIME FOR A RANDOM WALK ABOVE A RECEDING BOUNDARY

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Let $\xi_1, \xi_2, \dots$ be i.i.d. random variables with zero mean and finite variance $\sigma^2 := \mathbf{E}\xi_1^2 > 0$. Denote $S_k := \sum\limits_{i=1}^{k} \xi_i$, $k = 1, 2, ..., n$. We define the sojourn time for a trajectory of the random walk $\{S_1, \dots, S_n\}$ above a level $xg(\cdot)$ as the random variable

$$\tau_n(xg) := \sum_{k=1}^{n} I\{S_k \geq xg(k/n)\},$$

where $I(\cdot)$ is the indicator of an event, $x \equiv x(n) \to \infty$ as $n \to \infty$ characterizes the speed of the boundary moving, and a bounded positive function $g(t)$, $t \in (0, 1]$, determines the configuration of the curvilinear boundary in dependence on time.

In the first part of the talk, we study the asymptotics of the mean

$$\mathbf{E}\tau_n(xg) = \sum_{k=1}^{n} \mathbf{P}\{S_k \geq xg(k/n)\}$$

as $n \to \infty$ for a wide class of functions $g(\cdot)$ under Cramér's condition on the distribution of ξ_1.

Next we assume that $g(t) \equiv 1$. In the second part of the talk, we study the asymptotics of the tail probability $\mathbf{P}\{\tau_n(x)/n \geq y\}$ for any fixed $y \in (0, 1)$ in the case where $x \equiv x(n)$ tends to infinity in the moderate large deviations range. We assume that, for some $\lambda > 0$ and $r \in [1, 2]$,

1) $\mathbf{E}e^{\lambda|\xi_1|} < \infty$ and $\mathbf{E}\big\{e^{\lambda\xi_1^r} I(\xi_1 \geq 0)\big\} < \infty$;

2) $xn^{-1/2} \to \infty$ and $x = o\big(\min\big\{n^{(r+1)/(r+2)}, (n/\log n)^{3/4}\big\}\big)$.

Theorem. *Under conditions* 1 *and* 2, *for any fixed* $y \in (0, 1)$ *and* $n \to \infty$, *the following asymptotic relation is valid:*

$$\mathbf{P}\{\tau_n(x) \geq ny\} \sim \frac{2(1-y)^{3/2}}{\pi\sqrt{y}} \frac{n\sigma^2}{x^2} \exp\left\{-n(1-y)\Lambda\left(\frac{x}{n(1-y)}\right)\right\}, \quad (1)$$

where $\Lambda(z)$ *is the deviation function* (*Cramér's function*) *of* ξ_1:

$$\Lambda(z) := \sup_t \{tz - \log \psi_{\xi_1}(t)\}, \quad where \ \ \psi_{\xi_1}(t) := \mathbf{E}e^{t\xi_1};$$

in case $r = 1$, *the argument of the exponential function in* (1) *can be replaced with* $-x^2/(2\sigma^2 n(1-y))$, *and in case* $r \in (1,2]$, *with*

$$-\frac{x^2}{2\sigma^2 n(1-y)} + \frac{x^3 \mathbf{E}\xi_1^3}{6\sigma^6 n^2 (1-y)^2}.$$

Further we show that the statement of the theorem is valid in the deviation range $xn^{-1/2} \to \infty$ and $x = o((n/\log n)^{3/4})$ under Cramér's condition only (the case $r = 1$) if the component of distribution of ξ_1 on the positive half-line is exponential (or geometric in the discrete case). Another component of this distribution can be arbitrary at that (or arbitrary lattice-valued in the discrete case).

Notice that the asymptotic relation (1) is valid in the whole moderate large deviations range $xn^{-1/2} \to \infty$ and $x = o(n)$ for the simple symmetric random walk, i.e., if $\xi_1 = \pm 1$ with probabilities $1/2$. In this case,

$$\Lambda(z) = \frac{1+z}{2}\log(1+z) + \frac{1-z}{2}\log(1-z), \quad |z| < 1.$$

EXPONENTIAL INEQUALITIES FOR THE NUMBER OF TRIANGLES IN THE ERDÖS – RÉNYI RANDOM GRAPH

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Let $G(n,p)$ be the Erdös – Rényi graph, i.e. the random graph on n vertices where each edge is added independently with probability p. We obtain upper exponential inequalities for the tail probabilities of the centered and normalized number of triangles in $G(n,p)$. This characteristic has been extensively investigated by different authors. We highlight the paper of Janson, Oleszkiewicz and Rucinski (2004), where inequalities of this type are derived in case of general subgraphs. The order of the argument x in these inequalities coincide with our result, but our theorem also provides the exact constant:

Theorem. Let $T(n,p)$ be a number of triangles in $G(n,p)$. $b_n = \mathbb{D}T(n,p)$. Then for any x and $n \geq 7$

$$\mathbb{P}\left(b_n^{-1/2}|T(n,p) - \mathbb{E}T(n,p)| > x\right) \leq \exp\{-Cx^2\},$$

where

$$C = \frac{p^5(1-p)}{54\sqrt{2}e}.$$

A NEW TEST FOR THE ZIPF'S LAW

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We consider the classical multinomial occupancy scheme with infinitely many urns, where urns are numbered and balls are thrown one-by-one at random, each ball goes to urn $i = 1, 2, \dots$ with probability p_i. Here $\{p_i\}$ is a non-increasing sequence of strictly positive probabilities, with $\sum_i p_i = 1$. We let $X_{n,j}$ be the number of balls in urn j, out of the first n balls, and let $X_n = (X_{n,j},\ j = 1, 2, \dots)$.

The number of non-empty urns, $R_n = \#\{j : X_{n,j} > 0\}$, is a *measure of diversity* of the sample of size n. We will establish a new theoretically supported test for the Zipf's law. We introduce a new class of estimates that is based on the sequences $(R_1, \dots, R_n)$ and $(R'_1, \dots, R'_n)$, where $(R'_1, \dots, R'_n)$ are the sequences calculated from the back. We define an empirical process and prove its weak convergence to a centered Gaussian process. We calculate the covariance function of this limiting process. Then we construct a test of the omega-squared type. We will also discuss open problems and establish a number of auxiliary statements simplifying the study of these statistics.

MODIFICATIONS OF SIMON TEXT MODEL

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We discuss probability text models and their modifications. We prove the theorem on the convergence of a multidimensional process of the number of urns containing a fixed number of balls in the Simon model to a multidimensional Gaussian process. We introduce and investigate three two-parameter urn schemes that guarantee the power tail asymptotics of the number of occupied urns. These models do not follow the restriction on the limitation of the ratio of the number of urns with exactly one ball to the number of occupied urns that appears in the classical Bahadur-Karlin model.

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FAST MODE ESTIMATORS IN $\mathbb{R}^d$

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We will discuss the following algorithms for mode estimation in $\mathbb{R}^d$: the lattice-type estimator, the fraction-of-sample mode, the fraction-of-range mode, and the minimal variance mode. The lattice-type mode estimator has time complexity $O(dN)$, while, for the other listed estimators, there are variants with time complexity $O(dN^2)$, where N is the number of observations.

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IMPROVEMENT AND ANALYSIS OF STATISTICAL ESTIMATORS OF UNKNOWN PARAMETER IN ONE CLASS OF POWER REGRESSION PROBLEMS

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Suppose we observe random variables $\{Y_i\}$ represented in a way

$$Y_i = (1 + \alpha X_i)^r + \varepsilon_i, \quad i = 1, 2, ...,$$

where $\{X_i > 0\}$ is some known numerical sequence, $r \neq 0$ — a known number and $\{\varepsilon_i\}$ — i.i.d. unobservable random variables with

$$\mathbb{E}\varepsilon_1 = 0, \quad 0 < \mathrm{Var}(\varepsilon_1) = \sigma^2 < \infty.$$

Our aim is to estimate an unknown parameter $\alpha > 0$ using the first n observations.

Let α_n^* be the previously obtained consistent estimate of α. To improve it we propose, after R. Fisher's idea [1], using an estimate α_n^{**} in the form of

$$\alpha_n^{**}(c) = \alpha_n^* - \frac{S_{n,r}^{'}(\alpha_n^*)}{cS_{n,r}^{''}(\alpha_n^*) + (1-c)2r^2 E_n^2(\alpha_n^*)},$$

where c is a constant chosen by the statistician,

$$S_{n,r}(\alpha) = \sum_{i \leq n} (Y_i - (1 + \alpha X_i)^r)^2 \quad \text{and} \quad E_n^2(t) := \sum_{i \leq n} \frac{X_i^2}{(1 + tX_i)^{2-2r}}.$$

Under certain conditions it is proved that

$$\frac{\alpha_n^{**}(c) - \alpha}{d_n} \Rightarrow \eta,$$

for all c, where

$$\eta \sim \mathcal{N}(0, 1) \quad \text{and} \quad d_n = \frac{\sigma}{rE_n(\alpha)}.$$

The talk is based on a joint work [2] with Alexander I. Sakhanenko (Novosibirsk State University, Sobolev Institute of Mathematics, Novosibirsk).

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THE PROBABILITY OF EXCEEDING A HIGH BOUNDARY BY A HEAVY-TAILED BRANCHING RANDOM WALK IN VARYING ENVIRONMENT

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Consider a *branching process in varying environment* Z_n with sequence of offspring distributions $\widehat{\mathcal{P}}$ and genealogical tree $\mathcal{T}$. For arbitrary path π in $\mathcal{T}$ started in the root of $\mathcal{T}$ let

$$S(\pi) = \sum_{e \in \pi} \xi_{n(e),j(e)}$$

be a *branching random walk* (BRW), where $\{\xi_{n,j}\}$ is the sequence of i.i.d. zero-mean random variables (r.v.'s), which does not depend on branching mechanism (here $n(e)$ is the generation in which edge e ends, and $j(e)$ is the number of particle in generation n in which e ends). We assume in addition the branching mechanism does not depend on displacements of BRW and that $\xi_{1,1}$ has a *heavy right tail*, i.e. $\mathbb{E}e^{\lambda \xi_{1,1}} = \infty$ for all $\lambda > 0$.

For any non-negative function g on $\mathbb{Z}_+$ let

$$S^g(\pi) = S(\pi) - g(|\pi|)$$

be a *g-shifted* BRW. We obtain conditions for the relations

$$\mathbb{P}(R^g_\mu > x) \geq (1 + o(1)) H^g_\mu(x; \widehat{\mathcal{P}})$$

and

$$\mathbb{P}(R^g_\mu > x) = (1 + o(1)) H^g_\mu(x; \widehat{\mathcal{P}})$$

to hold uniformly over all suitable families of counting r.v.'s $\mu \leq \infty$ and functions g, where

$$H^g_\mu(x; \widehat{\mathcal{P}}) = \sum_{n=1}^{\infty} \mathbb{E}\left[Z_n \mathbb{I}(\mu \geq n)\right] \overline{F}(x + g(n)).$$

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