The probability of exceeding a high boundary by a heavy-tailed branching random walk in varying environment

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Presentation outline

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1 Branching Processes in Varying environment (BPVE)

2 Branching Random Walk (BRW)

BPVE

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- $\zeta_{n,1} \geq 1$ a.s. for any $n \geq 1$.

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$\mathcal{P}_0$ • $Z_0 = 1$

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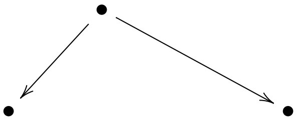

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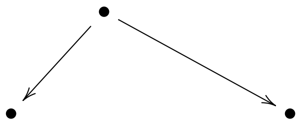

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Evolution of BPVE

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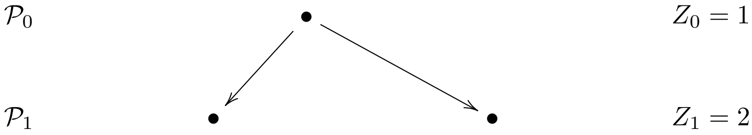

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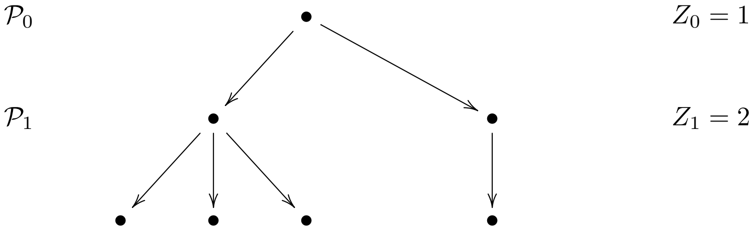

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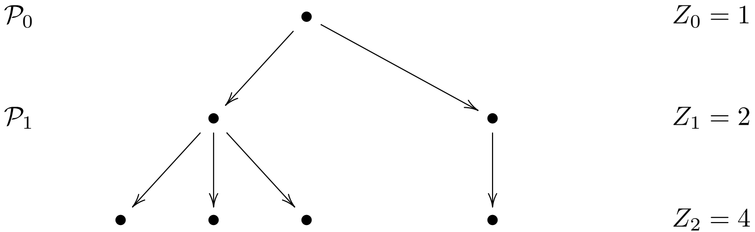

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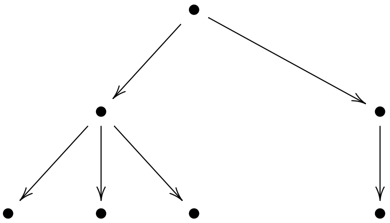

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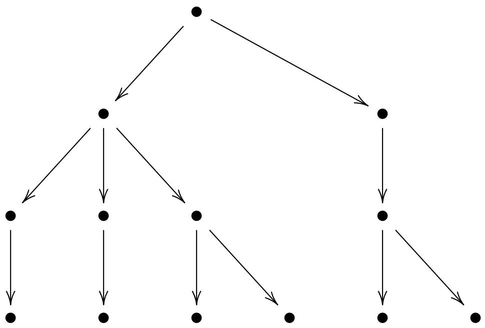

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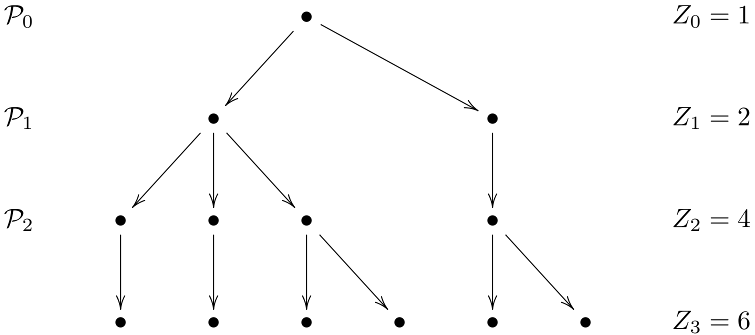

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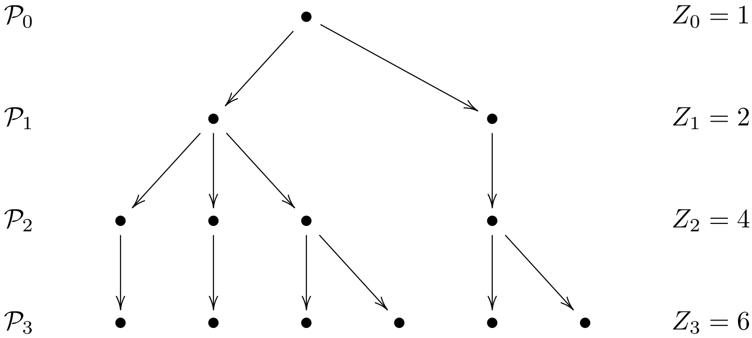

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$$\prod_{n=0}^{\infty} \mathbb{E}\zeta_1^{(n)} < \infty. \tag{1}$$

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Proposition 1

Let (1) holds. Then there exists $Z_\infty \in L_1(\Omega)$ such that

$$Z_n \to Z_\infty \text{ a.s. and in } L_1(\Omega) \text{ as } n \to \infty.$$

Moreover, the fading time

$$\nu = \inf\{n \geq 1 : Z_n = Z_{n+1} = \ldots = Z_\infty\} < \infty \quad \text{a.s.}$$

Properties of BPVE with fading

Goal: to obtain conditions for the finiteness of the moments of ν and Z_∞.

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BRW

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$\{Z_n\}$

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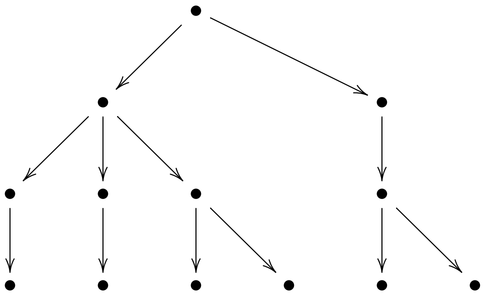
$$\{Z_n\} \leftrightarrow \mathcal{T} = (\mathcal{V}, \mathcal{E})$$



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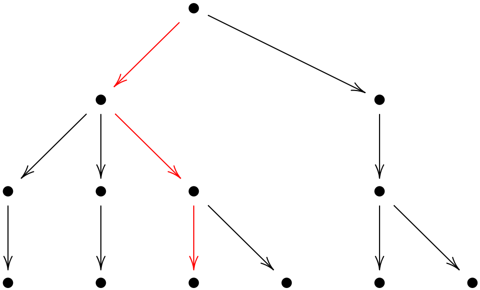

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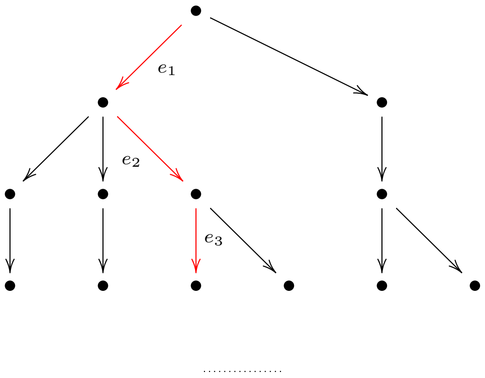

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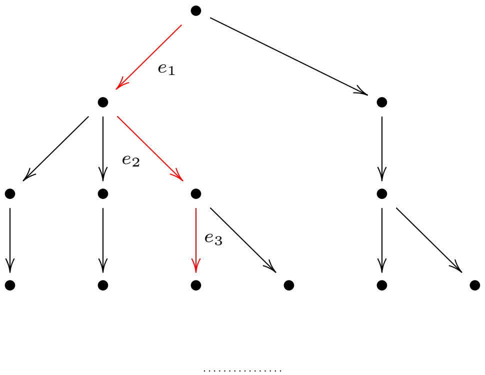

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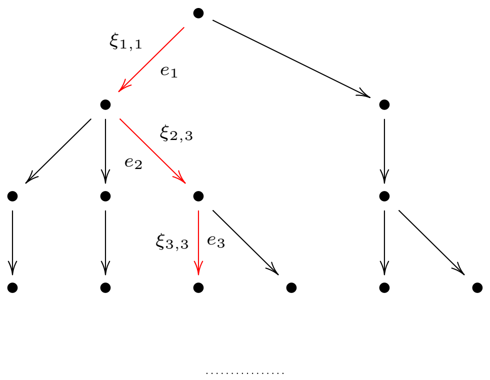

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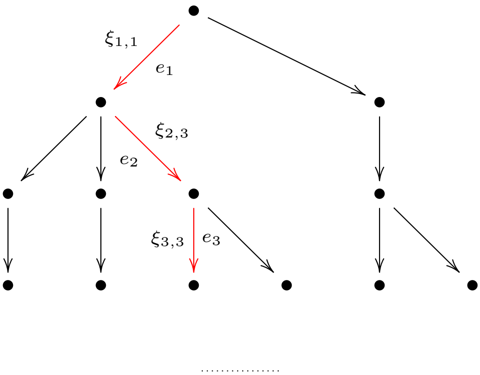

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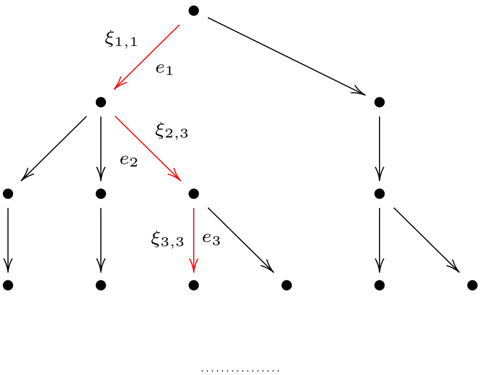

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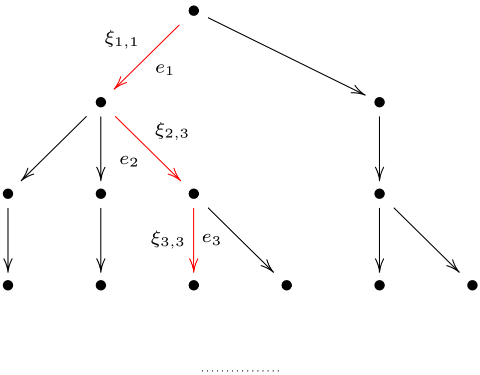

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g-shifted BRW:

$$S^g(\pi) = \sum_{e \in \pi} \xi_{n(e),j(e)} - g(|\pi|)$$

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$$\mathbb{P}\left(R_\mu^g > x\right) \sim ? \text{ as } x \to \infty$$

where $\mu \le \infty$ is the counting r.v.