Exponential inequalities for the number of triangles in the Erdös-Rényi random graph

Alexander Bystrov Novosibirsk State University Nadezhda Volodko Sobolev Institute of Mathematics

August 26, 2020

イロト イ理ト イヨト イヨト







Alexander Bystrov, Nadezhda Volodko Exponential inequalities for the number of triangles

イロト イポト イヨト イヨト







Alexander Bystrov, Nadezhda Volodko Exponential inequalities for the number of triangles

イロン イロン イヨン イヨン

Random field

Let $\{\xi_{ijk}\}_{i < j < k}$ be a field of bounded random variables, not necessarilly identically distributed. Two different elements of the field are independent if at least two indices of the triples do not coincide.

ヘロン 人間 とくほ とくほ とう

-

Let $\{\xi_{ijk}\}_{i < j < k}$ be a field of bounded random variables, not necessarilly identically distributed. Two different elements of the field are independent if at least two indices of the triples do not coincide.

If exactly two indices of the triples coincide then the covariance is separated from zero:

$$\operatorname{cov}(\xi_{ijk},\xi_{ijl}) \ge \rho > \mathbf{0}, \ k \neq l; \ \mathbb{D}\xi_{ijk} = \sigma^2 > \mathbf{0}; \ |\xi_{ijk}| \le C \ a.s. \ (1)$$

く 同 と く ヨ と く ヨ と

The object

The object of our interest is the centered and normalised sum

$$R_n = b_n^{-1/2}(T_n - \mathbb{E}T_n),$$

where

$$T_n = \sum_{1 \le i < j < k \le n} \xi_{ijk};$$

ヘロト ヘワト ヘビト ヘビト

The object

The object of our interest is the centered and normalised sum

$$R_n=b_n^{-1/2}(T_n-\mathbb{E}T_n),$$

where

$$T_n = \sum_{1 \le i < j < k \le n} \xi_{ijk};$$
$$b_n = \mathbb{D}T_n \ge \sigma^2 \binom{n}{3} + \rho \frac{P_{n,4}}{2}.$$

イロト イポト イヨト イヨト

l

The object

The object of our interest is the centered and normalised sum

$$R_n=b_n^{-1/2}(T_n-\mathbb{E}T_n),$$

where

$$T_n = \sum_{1 \le i < j < k \le n} \xi_{ijk};$$

$$b_n = \mathbb{D}T_n \ge \sigma^2 \binom{n}{3} + \rho \frac{P_{n,4}}{2}.$$

Our aim is obtaining Hoeffding-type inequalities for the distribution tails of R_n with the explicit constant in the exponent.

・ 同 ト ・ ヨ ト ・ ヨ ト

The number of triangles in the Erdös-Rényi graph

The most natural example of T_n is the number of triangles in the Erdös-Rényi graph, i.e. the random graph on *n* vertices where each edge is added independently with probability *p*.

・ 同 ト ・ ヨ ト ・ ヨ ト …

The number of triangles in the Erdös-Rényi graph

The most natural example of T_n is the number of triangles in the Erdös-Rényi graph, i.e. the random graph on *n* vertices where each edge is added independently with probability *p*. Here

$$\xi_{ijk} = X_{ij}X_{jk}X_{ki},$$

where independent $X_{ij} \in \mathbb{B}_{\rho}$,

< 回 > < 回 > < 回 > -

The number of triangles in the Erdös-Rényi graph

The most natural example of T_n is the number of triangles in the Erdös-Rényi graph, i.e. the random graph on *n* vertices where each edge is added independently with probability *p*. Here

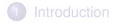
$$\xi_{ijk} = X_{ij}X_{jk}X_{ki},$$

where independent $X_{ij} \in \mathbb{B}_{\rho}$,

$$\sigma^2 = p^3(1-p^3), \ \operatorname{cov}(\xi_{ijk},\xi_{ijl}) = \rho = p^5(1-p), \ k \neq l, \ C = 1.$$

・ 同 ト ・ ヨ ト ・ ヨ ト …







Alexander Bystrov, Nadezhda Volodko Exponential inequalities for the number of triangles

イロト イポト イヨト イヨト

History for the number of triangles

- CLT A. Rucinski (1988)
- Large deviations S. Chatterjee and S.R.S. Varadhan (2011) and others
- Exponential inequalities S. Janson, K. Oleszkiewicz, A. Rucinski (2002)

ヘロン 人間 とくほ とくほ とう

э.

History for the number of triangles

- CLT A. Rucinski (1988)
- Large deviations S. Chatterjee and S.R.S. Varadhan (2011) and others
- Exponential inequalities S. Janson, K. Oleszkiewicz, A. Rucinski (2002)

Let us cite the result of S. Janson, K. Oleszkiewicz and A. Rucinski concerning exponential inequalities for the distribution tails of the number of triangles:

イロト イ押ト イヨト イヨトー

1

History for the number of triangles

- CLT A. Rucinski (1988)
- Large deviations S. Chatterjee and S.R.S. Varadhan (2011) and others
- Exponential inequalities S. Janson, K. Oleszkiewicz, A. Rucinski (2002)

Let us cite the result of S. Janson, K. Oleszkiewicz and A. Rucinski concerning exponential inequalities for the distribution tails of the number of triangles:

$$\mathbb{P}(T_n \geq \mathbb{E}T_n + \varepsilon n^3 p^3) \leq \exp(-\alpha(\varepsilon)n^2 p^2).$$

 $\alpha(\varepsilon)$ has no explicit form here.

イロト イ押ト イヨト イヨトー

Theorem

Theorem

Under conditions (1) for $n \ge 7$ the following upper estimate holds:

$$\mathbb{P}(|R_n| > x) \le \exp\left\{-\frac{1}{2e}\left(\frac{x}{C_0}\right)^2\right\},$$
(2)

where

$$C_0 = C \left(\frac{27\sqrt{2}}{\rho}\right)^{1/2}$$

◆□ ▶ ◆□ ▶ ◆ □ ▶ ◆ □ ▶ ◆ □ ● ○ ○ ○