Exponential inequalities for the number of triangles in the Erdös-Rényi random graph

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Outline

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If exactly two indices of the triples coincide then the covariance is separated from zero:

$$\operatorname{cov}(\xi_{ijk}, \xi_{ijl}) \geq \rho > 0,\ k \neq l;\ \ \mathbb{D}\xi_{ijk} = \sigma^2 > 0;\ \ |\xi_{ijk}| \leq C\ \ a.s.\quad (1)$$

The object

The object of our interest is the centered and normalised sum

$$R_n = b_n^{-1/2}(T_n - \mathbb{E}T_n),$$

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Our aim is obtaining Hoeffding-type inequalities for the distribution tails of R_n with the explicit constant in the exponent.

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where independent $X_{ij} \in \mathbb{B}_p$,

$$\sigma^2 = p^3(1 - p^3), \;\; \operatorname{cov}(\xi_{ijk}, \xi_{ijl}) = \rho = p^5(1 - p), \; k \neq l, \;\; C = 1.$$

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- CLT — A. Rucinski (1988)
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$$\mathbb{P}(T_n \geq \mathbb{E}T_n + \varepsilon n^3 p^3) \leq \exp(-\alpha(\varepsilon) n^2 p^2).$$

$\alpha(\varepsilon)$ has no explicit form here.

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Under conditions (1) for $n \geq 7$ the following upper estimate holds:

$$\mathbb{P}(|R_n| > x) \leq \exp\left\{ - \frac{1}{2e}\left(\frac{x}{C_0}\right)^2 \right\}, \qquad (2)$$

where

$$C_0 = C\left(\frac{27\sqrt{2}}{\rho}\right)^{1/2}.$$