# Parisian ruin with random deficit-dependent delays for spectrally negative Lévy processes

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- Dassios and Wu (2008): "Parisian type ruin will occur if the surplus falls below zero and stays below zero for a continuous time interval of length $d$. In some respects, this might be a more appropriate measure of risk than classical ruin as it gives the office some time to put its finances in order."

- The time period during which the surplus is allowed to remain negative: [implementation] delay (or grace) period.

- The idea & name go back to *Parisian options*: payoff depends on the lengths of the excursions of the underlying asset price above or below a barrier (e.g., a *Parisian down-and-out option* expires if the underlying price drops below a given level and stays constantly below that level for longer than $d$).

- Stopping times of this kind were first considered by Chesney, Jeanblanc-Picque and Yor (1997).
- The idea appeared in the actuarial literature even earlier. E.g., Dos Reis (1993): if a company has many portfolios and only one of them $< 0$, it can have enough funds (either from another line of business or as a loan from a bank) to support the affected portfolio for some time.
- More recently: Parisian ruin $\approx$ theor'l descr'n of reorganization under Ch. 11 of the US Bankruptcy Code of a company in distress rather than its immediate liquidation under Ch. 7. Ch. 11 allows the company to control its operations with a bankruptcy court oversight. The court grants the company an observation period.

When the delay period length $d = \text{const}$ (and fixed):

- Dassios and Wu (2008) derived the Laplace transform of the time until the Parisian ruin & the probability thereof for the classical Cramér–Lundberg (CL) model.

- Loeffen, Czarna, Palmowski (2013): an elegant compact formula for the Parisian ruin prob'ty in the case when the risk process $X = \{X_t\}_{t \geq 0}$ is a spectrally negative Lévy process (SNLP) (in terms of the scale function of $X$ and the distribution of $X_d$).

- Czarna (2016): in the SNLP framework, Parisian ruin prob's with an "ultimate bankruptcy level", when ruin will also occur if the deficit reaches a given deterministic negative level. Simpler proofs & further results: Czarna & Renaud (2016).

When the delay period length is random:

- Landriault, Renaud & Zhou (2014), SNLP (bdd variation), delay times are i.i.d.: Laplace transform of the Parisian ruin time when delays were exp–distributed or followed Erlang mixture distributions (NB: switching to stochastic delays with such distr'ns improves the tractability).

- Frostig and Keren-Pinhasik (2020): studied Parisian ruin with ultimate bankruptcy barrier for i.i.d. exp– and Erlang–distributed random delays.

- Baurdoux, Pardo, Pérez & Renaud (2016) studied the Gerber-Shiu distribution at Parisian ruin with exp-distributed delays in the SNLP setup.

In the present talk: a natural interesting extension to the Parisian ruin problem with a risk reserve SNLP, where the distr'n of the random delay lengths **can depend on the deficit** at the epochs when the risk reserve process turns negative, starting a new negative excursion. This includes the possibility of an immediate ruin when the deficit hits a certain subset.

In this general setting, we derive a closed-from expression for the Parisian ruin probability and the joint Laplace transform of the Parisian ruin time and the deficit at ruin.

Examples: the risk reserve follows the classical CL dynamics, whereas the delay period distr'ns are finite mixtures of Erlang distr'ns with parameters depending on the deficit value at the beginning of the respective negative excursion.

- $X := \{X_t\}_{t\geq 0}$ is an SNLP with càdlàg paths, starting at $X_0 = u \in \mathbb{R}$ (we use: $\mathbf{P}_u$, $\mathbf{E}_u$).
- First assume: trajectories are of locally bounded variation. The cumulant generating function $\psi(\theta) := \ln \mathbf{E}_0 e^{\theta X_1}$ of $X$:

$$\psi(\theta) := a\theta + \int_{(-\infty,0)} (e^{\theta x} - 1)\Pi(dx), \quad \theta \geq 0,$$

  where the measure $\Pi$ is such that $\int_{(-1,0)} |x|\Pi(dx) < \infty$.
- Our $X$ is just a linear drift minus a pure jump subordinator.
- Also assume satisfied the standard safety loading condition

$$\mathbf{E}_0 X_1 > 0 \tag{1}$$

  (clearly, $\mathbf{E}_0|X_1| < \infty$ under the above condition as $X$ is SN).

Denote by $\mathbb{F} := \{\mathcal{F}_t\}_{t\geq 0}$ the natural filtration for $X$. For $x, y \in \mathbb{R}$, introduce the first hitting times

$$\tau_x^- := \inf\{t > 0 : X_t < x\} \quad \text{and} \quad \tau_y^+ := \inf\{t > 0 : X_t > y\}.$$

In view of (1), $\tau_x^-$ is an improper random variable when $x \leq X_0$.

Setting $\tau_{0,0}^+ := 0$, we further define recursively for $k = 1, 2, \ldots$ the following (improper, due to (1)) $\mathbb{F}$-stopping times:

$$\tau_{0,k}^- := \inf\{t > \tau_{0,k-1}^+ : X_t < 0\} \quad \text{and} \quad \tau_{0,k}^+ := \inf\{t > \tau_{0,k}^- : X_t > 0\}.$$

NB: due to (1), the time $\tau_{0,k}^+$ is always finite on the event $\{\tau_{0,k}^- < \infty\}$.

If $\tau_{0,k-1}^- < \infty$ but $\tau_{0,k}^- = \infty$ for some $k \geq 1$, then there are exactly $k - 1$ negative excursions of the risk reserve process.

To construct random delay times: assume $P_x(B)$ is a stoch. kernel on $(-\infty, 0) \times \mathcal{B}([0,\infty))$: $\forall B \in \mathcal{B}([0,\infty))$, $P_x(B)$ is a measurable function of $x$; $\forall x < 0$, $P_x(B)$ is a probability measure in $B \in \mathcal{B}([0,\infty))$.

Let $F_x(s) := P_x((-\infty, s])$, $s \ge 0$, be the DF of $P_x$, $\overline{F}_x(s) := 1 - F_x(s)$,

$$F_x^{\leftarrow}(y) := \inf\{s \ge 0 : F_x(s) \ge y\}, \ y \in (0,1), \text{ the gen'd inverse of } F_x.$$

NB: $F_x^{\leftarrow}(y)$, $(x,y) \in D := (-\infty, 0) \times (0,1)$, is a measurable function.

Let $\{U_n\}_{n\ge1}$ be i.i.d. $U(0,1)$, independent of $X$. The length $\eta_k$ of the $k$-th delay window, $k = 1, 2, \ldots$, is then defined on $\{\tau_{0,k}^- < \infty\}$ as

$$\eta_k := F_{\chi_k}^{\leftarrow}(U_k), \quad \text{where} \ \ \chi_k := X_{\tau_{0,k}^-}$$

(on $\{\tau_{0,k}^- = \infty\}$ we can leave both $\chi_k$ and $\eta_k$ undefined).

This includes situations where $\eta_k = 0$ for some values of $\chi_k$.

We say that Parisian ruin occurs in our model if

$$N := \inf\{k \geq 1 : \tau_{0,k}^{-} < \infty, \tau_{0,k}^{-} + \eta_k < \tau_{0,k}^{+}\} < \infty,$$

and define on the event $\{N < \infty\}$ the Parisian ruin time as

$$T := \tau_{0,N}^{-} + \eta_N.$$

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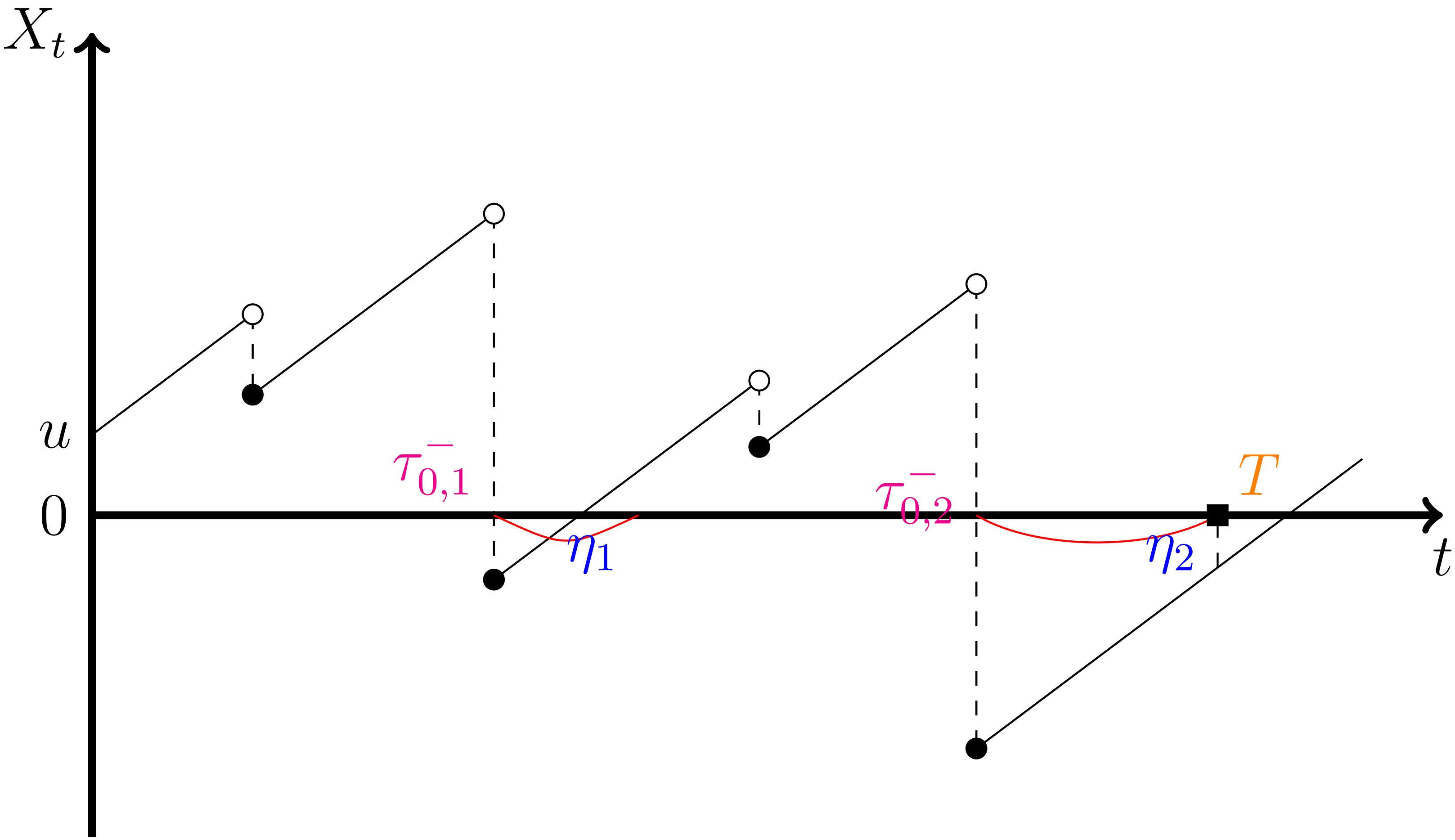



Figure 1: Parisian ruin occurs at time $T$ during the second negative excursion ($N = 2$) as the recovery time exceeded the window length.

For $q \geq 0$, the $q$-scale function $W^{(q)}$ for $X$ is a function on $\mathbb{R}$ s.t. (i) $W^{(q)}(x) = 0$ for $x < 0$ and (ii) $W^{(q)}(x)$ is continuous on $[0, \infty)$ and

$$\int_{[0,\infty)} e^{-\beta x} W^{(q)}(x) dx = \frac{1}{\psi(\beta) - q}, \quad \beta > \Phi(q),$$

where $\Phi(q) := \sup\{\theta \geq 0 : \psi(\theta) = q\}$, $q \geq 0$

One refers to $W := W^{(0)}$ as just the scale function for $X$.

The $q$-scale functions can be obtained as the scale functions for SNLPs with "tilted distributions": for $q \geq 0$,

$$W^{(q)}(x) = e^{\Phi(q)x} W_{\Phi(q)}(x), \quad x \in \mathbb{R},$$

where $W_\nu(x)$ is the scale function for the Lévy process with the cumulant function $\psi_\nu(\theta) := \psi(\theta + \nu) - \psi(\nu)$.

Several important characteristics of and fluctuation identities for SNLPs can be expressed in terms of their scale functions.

In particular, the distr'n $\mathbf{P}_u(\chi_1 \in \cdot\, , \tau_0^- < \infty)$ of the first negative value $\chi_1$ of $X$ given $X_0 = u > 0$ has (defective) density

$$h_u(x) = \int_{(0-,u]} \Pi((-\infty, x+z-u])dW(z), \quad x < 0.$$

Another one: for $q \geq 0$ and $t, y > 0$,

$$\mathbf{E}_0(e^{-q\tau_y^+}; \tau_y^+ \leq t) = e^{-qt}\Lambda^{(q)}(-y, t), \tag{2}$$

where

$$\Lambda^{(q)}(x,t) := \int_0^\infty W^{(q)}(x+z)\frac{z}{t}\mathbf{P}_0(X_t \in dz), \quad x \in \mathbb{R},\ t > 0.$$

Finding a closed-form expression for the scale function is a non-trivial problem.

A “robust” numerical method for computing $W^{(q)}$ was described in Surya (2008), whereas Egami & Yamazaki (2014) presented a possible “phase-type-fitting approach” to approximating scale functions, and Hubalek & Kyprianou (2010) presented several examples where closed form expressions for the scale function are available and described a methodology for finding such expression.

Further notation:

$$G_y(t) := \mathbf{P}_0(\tau_y^+ \le t) = -y \frac{\partial}{\partial y} \int_0^t \mathbf{P}_0(X_s > y) \frac{ds}{s}, \quad y, t > 0,$$

where the expression on the RHS comes from Kendall's formula,

$$K(x) := \mathbf{E}_0 \overline{F}_x(\tau_{|x|}^+) = \int_0^\infty \overline{F}_x(t) dG_{|x|}(t), \quad x < 0,$$

$$H(v) := \int_{-\infty}^0 K(x) h_v(x) dx, \quad v \ge 0.$$

**Theorem 1** *Under the above assumptions, the probability of no Parisian ruin when the initial reserve is $X_0 = u \geq 0$ is equal to*

$$\mathbf{P}_u(N = \infty) = \mathbf{E}_0 X_1 \left( W(u) + \frac{W(0)}{1 - H(0)} H(u) \right). \tag{3}$$

The probability of no "usual ruin" given $X_0 = u \geq 0$ is $\mathbf{E}_0 X_1 W(u)$.

To state more results, need more notations. For $v, w \geq 0$, $x < 0$, set

$$M_1(v,w,x) := \int_0^1 \Big[e^{(\psi(w)-v)F_x^{\leftarrow}(s)+wx} - e^{-vF_x^{\leftarrow}(s)}\Lambda^{(\psi(w))}(x, F_x^{\leftarrow}(s))\Big]\,ds,$$

$$M_2(v,x) := \int_0^1 e^{-vF_x^{\leftarrow}(s)}\Lambda^{(v)}(x, F_x^{\leftarrow}(s))\,ds.$$

Finally, assuming in addition that $u \in [0,b]$, $b > 0$, we set

$$Q_1(u,v,w) := \int_0^b \int_{(-\infty,-y)} M_1(v,w,y+\theta)\Big(\frac{W^{(v)}(u)W^{(v)}(b-y)}{W^{(v)}(b)} - W^{(v)}(u-y)\Big)\Pi(d\theta)dy,$$

$$Q_2(u,v) := \int_0^b \int_{(-\infty,-y)} M_2(v,y+\theta)\Big(\frac{W^{(v)}(u)W^{(v)}(b-y)}{W^{(v)}(b)} - W^{(v)}(u-y)\Big)\Pi(d\theta)dy.$$

NB: all computable once you got access to the scale functions.

**Theorem 2** *Under the above assumptions, for* $b, v, w \geq 0$ *and* $u \in [0, b]$*, one has*

$$\mathbf{E}_u(e^{-vT+wX_T}; T < \tau_b^+) = Q_1(u, v, w) + \frac{Q_1(0, v, w)Q_2(u, v)}{1 - Q_2(0, v)}.$$

Can compute $\mathbf{E}_u(e^{-vT+wX_T}; T < \infty)$ (a bit simpler expression).

Now what to do if *trajectories are of unbounded variation*?

The recursive procedure used does not work! For non-random delays of length $d > 0$, the Parisian ruin time was defined in Dassios & Wu (2008) as

$$T_d := \inf\{t > 0 : t - g_t > d\}, \quad \text{where} \ \ g_t := \sup\{s \in [0,t] : X_s \geq 0\},$$

using the convention that $\inf \varnothing = \infty$, $\sup \varnothing = 0$.

In the case of bounded variation trajectories and a common degenerate distribution $F_x(t) = \mathbf{1}(d \leq t)$, $x < 0$, for delay windows, the thus defined $T_d$ coincides with our $T$.

Extending this to the case of random delay windows is non-trivial. An approach to doing this in the simple situation where all the delay windows are i.i.d. exp'l was suggested in Baurdoux et al. (2009): denote by $G$ the set of all left-end points of the negative excursions of the process $X$ and then, "for each $g \in G$," consider "an independent, exponentially distributed RV $\mathbf{e}_q^g$, also independent of $X$" ($q$ represents here the rate of the exponential distribution).

The time of the Parisian ruin with i.i.d. exponentially distributed delay windows was defined as

$$\inf\{t > 0 : X_t < 0 \text{ and } t - g_t > \mathbf{e}_q^{g_t}\}.$$

Requires clarification with regard to exactly how these random times $\mathbf{e}_q^g$ are to be constructed.

Moreover, from the practical viewpoint, this is hardly meaningful as the mechanism is not feasible: if, say, $X_t = X_0 + ct + \sigma B_t$, $t \geq 0$, where $B$ is the std BM process then, immediately after the start of the first negative excursion at time $\tau_0^-$, one would have to generate infinitely many i.i.d. exp'l random times as the process $X$ will have infinitely many negative excursions in any right neighborhood of $\tau_0^-$.

To avoid complications and end up with an implementable Parisian-type ruin scheme: consider "$\varepsilon$-Parisian ruin times" $T^\varepsilon$ constructed for $\varepsilon > 0$ by "activating the clock" for random delay windows at the times when the value of $X_t$ drops below $-\varepsilon$ (such times were considered in Loeffen et al. (2013), Bardoux et al. (2016) as well). This makes it possible to use the recursive procedure we used for processes with trajectories of bounded variation.

To “recover”, the process needs to return to $[0,\infty)$.

**Theorem 3** *Under the above assumptions, in the case of a general SNLP, the probability of no $\varepsilon$-Parisian ruin when the initial reserve is $X_0 = u \geq 0$ is equal to*

$$\mathbf{P}_u(N^\varepsilon = \infty) = \mathbf{E}_0 X_1 \left[ W(u+\varepsilon) + \frac{W(\varepsilon)\mathbf{E}_{u+\varepsilon}(K(\chi_1 - \varepsilon); \tau_0^- < \infty)}{1 - \mathbf{E}_\varepsilon(K(\chi_1 - \varepsilon); \tau_0^- < \infty)} \right].$$

How to prove Theorem 1? The first step is similar to the one in Loeffen et al. (2013):

$$\begin{aligned}
\mathbf{P}_u(N=\infty) &= \mathbf{P}_u(\tau_0^- = \infty) + \mathbf{P}_u(\tau_0^- < \infty, N = \infty) \\
&= \mathbf{P}_u(\tau_0^- = \infty) + \mathbf{E}_u\mathbf{E}_u\big(\mathbf{1}(\tau_0^- < \infty)\mathbf{1}(N=\infty)|\mathcal{F}_{\tau_0^-}\big) \\
&= \mathbf{P}_u(\tau_0^- = \infty) + \mathbf{E}_u\big[\mathbf{1}(\tau_0^- < \infty)\mathbf{E}_u\big(\mathbf{1}(N=\infty)|\mathcal{F}_{\tau_0^-}\big)\big].
\end{aligned}$$

By the strong Markov property and the absence of positive jumps, on the event $\{\tau_0^- < \infty\}$ the process $\widetilde{X} := \{\widetilde{X}_t := X_{\tau_{0,1}^+ + t}\}_{t\geq 0}$ is an independent of $\mathcal{F}_{\tau_{0,1}^+}$ Lévy process with the same cumulant as $X$, but with initial value $\widetilde{X}_0 = 0$.

For the conditional expectation, on $\{\tau_0^- < \infty\}$:

$$\begin{aligned}
\mathbf{E}_u\big(\mathbf{1}(N=\infty)|\mathcal{F}_{\tau_0^-}\big) &= \mathbf{E}_u\big(\mathbf{1}(\tau_0^- + \eta_1 \geq \tau_{0,1}^+)\mathbf{1}(\widetilde{N}=\infty)|\mathcal{F}_{\tau_0^-}\big) \\
&= \mathbf{E}_u\big[\mathbf{E}_u\big(\mathbf{1}(\tau_0^- + \eta_1 \geq \tau_{0,1}^+)\mathbf{1}(\widetilde{N}=\infty)|\mathcal{F}_{\tau_{0,1}^+}\big)\big|\mathcal{F}_{\tau_0^-}\big] \\
&= \mathbf{P}_0(N=\infty)\mathbf{E}_u\big[\mathbf{E}_u\big(\mathbf{1}(\eta_1 \geq \tau_{0,1}^+ - \tau_0^-)|\mathcal{F}_{\tau_{0,1}^+}\big)\big|\mathcal{F}_{\tau_0^-}\big],
\end{aligned}$$

where the inner conditional expectation is

$$\begin{aligned}
\mathbf{E}_u\big(\mathbf{1}(\eta_1 \geq \tau_{0,1}^+ - \tau_0^-)|\mathcal{F}_{\tau_{0,1}^+}\big) &= \mathbf{E}_u\big[\mathbf{1}(U_1 \geq F_{\chi_1}(\tau_{0,1}^+ - \tau_0^-))|\mathcal{F}_{\tau_{0,1}^+}\big] \\
&= \overline{F}_{\chi_1}(\tau_{0,1}^+ - \tau_0^-).
\end{aligned}$$

Can show:

$$\mathbf{E}_u\big(\overline{F}_{\chi_1}(\tau_{0,1}^+ - \tau_0^-)\big|\mathcal{F}_{\tau_0^-}\big) = K(\chi_1),$$

End up with:

$$\mathbf{P}_u(N = \infty) = \mathbf{P}_u(\tau_0^- = \infty) + \mathbf{P}_0(N = \infty)\mathbf{E}_u(K(\chi_1); \tau_0^- < \infty).$$

Setting now $u = 0$ yields

$$\mathbf{P}_0(N = \infty) = \frac{\mathbf{P}_0(\tau_0^- = \infty)}{1 - \mathbf{E}_0(K(\chi_1); \tau_0^- < \infty)}.$$

Plugging this in the previous formula and expressing the expectations in terms of our function $H$ completes the proof. $\square$

Proving Theorem 2 is more fun (and work). See `arXiv`. $\square$

**Examples.** Consider the classical CL model:

$$X_t = X_0 + ct - \sum_{j=1}^{A_t} \xi_j, \quad t \geq 0,$$

where $c > 0$ is a constant premium payment rate, the Poisson claims arrival process $\{A_t\}_{t\geq 0}$ with rate $\lambda > 0$ is independent of i.i.d. exp–distributed claim sizes $\{\xi_n\}_{n\geq 1}$ with rate $\alpha > 0$.

Here $\psi(\theta) = c\theta + \lambda(\frac{\alpha}{\alpha+\theta} - 1)$, $\theta > -\alpha$, so that condition (1) turns into

$$\mathbf{E}_0 X_1 = c - \lambda/\alpha > 0.$$

A well-known result:

$$\mathbf{P}_u(\tau_0^- < \infty) = \frac{\lambda}{\alpha c} e^{-(\alpha-\lambda/c)u}, \quad u \geq 0.$$

Elementary computation yields

$$\Phi(q) = \frac{1}{2c}\Big(\sqrt{(\alpha c - \lambda - q)^2 + 4q\alpha c} - (\alpha c - \lambda - q)\Big), \quad q \geq 0,$$

$$W(x) = \frac{\alpha}{\alpha c - \lambda}\Big(1 - \frac{\lambda}{\alpha c} e^{-(\alpha - \lambda/c)x}\Big)\mathbf{1}(x \geq 0), \quad \text{with } \; W(0) = \frac{1}{c}.$$

An series expr'n for $W^{(q)}$ is available in Behme & Oechsler (2020).

As claims are exp'l, one has $h_u(x) = \frac{\lambda}{c} e^{\alpha x - (\alpha - \lambda/c)u}$, $x < 0$, and hence

$$H(v) = \int_{-\infty}^{0} K(x) h_v(x)\, dx = H(0) e^{-(\alpha - \lambda/c)v}.$$

Now it follows from Theorem 1 that

$$\mathbf{P}_u(N < \infty) = \frac{\lambda}{\alpha c}\left[1 - \frac{(\alpha c - \lambda)H(0)}{\lambda(1 - H(0))}\right] e^{-(\alpha - \lambda/c)u}, \quad u \geq 0.$$

Need: $H(0) = \frac{\lambda}{c}\int_{-\infty}^{0} e^{\alpha x} K(x)dx$, $K(x) = \int_0^\infty \overline{F}_x(t)dG_{|x|}(t)$, $x < 0$.

Let $\overline{F}_x(t) = e^{-r(x)t}$, $t > 0$ for some Borel $r : (-\infty, 0) \to (0, \infty]$ ($r(x) = \infty$ means immediate ruin when $\chi_1$ is equal to that $x$).

Known: $K(x) = e^{\Phi(r(x))x}$, $x < 0$, so $H(0) = \frac{\lambda}{c}\int_{-\infty}^{0} e^{[\alpha+\Phi(r(x))]x}dx$.

Can be evaluated, e.g., when $r(x)$ is p/w constant: for some $n \geq 1$, $r_k \in (0, \infty]$, $k = 1, \ldots, n$, and $-\infty =: a_0 < a_1 < \cdots < a_{n-1} < a_n := 0$,

$$r(x) := \sum_{k=1}^{n} r_k \mathbf{1}(x \in (a_{k-1}, a_k]).$$

Then

$$H(0) = \frac{\lambda}{c}\sum_{k=1}^{n}\int_{a_{k-1}}^{a_k} e^{(\alpha+\Phi(r_k))x}dx = \frac{\lambda}{c}\sum_{k=1}^{n}\frac{e^{(\alpha+\Phi(r_k))a_k} - e^{(\alpha+\Phi(r_k))a_{k-1}}}{\alpha+\Phi(r_k)}.$$

Suppose the conditional distribution of the window length is a finite mixture of Erlang distr'ns with parameters depending on the deficit: for an $m \geq 1$, there are Borel functions $p_j : (-\infty, 0) \to [0,1]$, $\sum_{j=1}^m p_j(x) \equiv 1$, $r_j : (-\infty, 0) \to (0, \infty]$, and $\nu_j(x) : (-\infty, 0) \to \mathbb{N}$, $j = 1, \ldots, m$, such that, for $x < 0$,

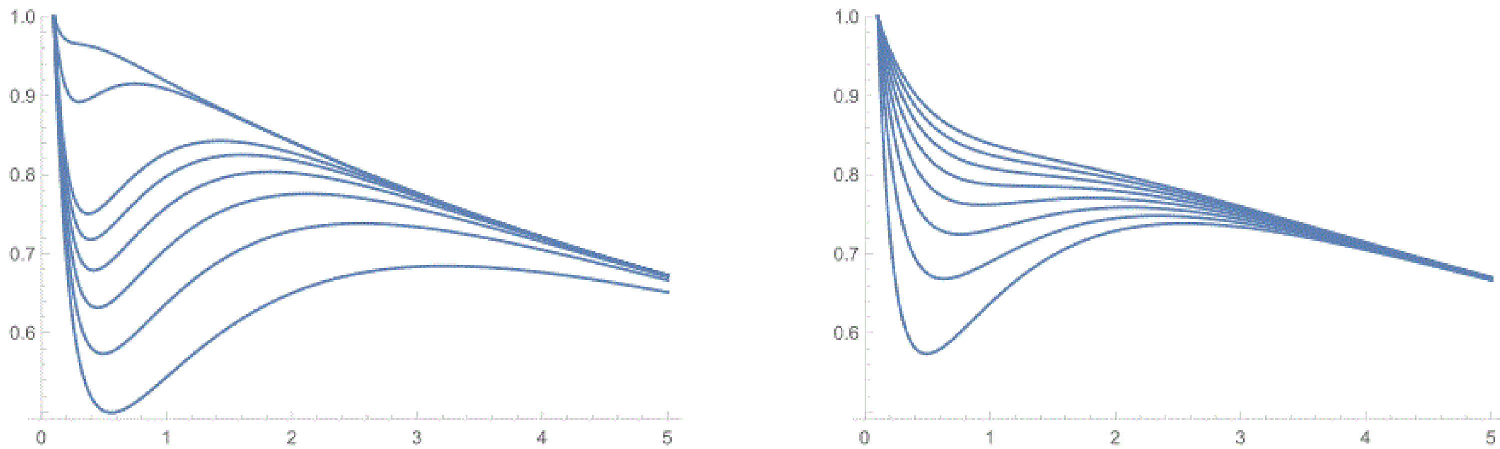
$$\overline{F}_x(t) = \sum_{j=1}^m p_j(x) \sum_{\ell=0}^{\nu_j(x)-1} \frac{(r_j(x)t)^\ell}{\ell!} e^{-r_j(x)t}, \quad t > 0.$$

Such mixtures form a rather large class: it is well-known to be everywhere dense in the weak convergence topology in the class of continuous distributions on $(0, \infty)$.

If we further assume, as above, that the functions participating in the definition of $\overline{F}_x$ are piece-wise constant, we can also get an explicit expression for $H(0)$:

$$H(0) = \frac{\lambda}{c} \sum_{k=1}^{n} \sum_{j=1}^{m} p_{j,k} \sum_{\ell=0}^{\nu_{j,k}-1} r_{j,k}^{\ell} \int_{a_{k-1}}^{a_k} e^{\alpha x} \phi_\ell(r_{j,k}, x) dx,$$

where $\phi_\ell(r, x) := \frac{(-1)^\ell}{\ell!} \frac{\partial^\ell}{\partial r^\ell} e^{\Phi(r)x}$.




(a) $a_1 = -0.75$ (the bottom curve); $-1$; $-1.25$; $-1.5$; $-1.75$; $-2$; $-4$; $-8$ (the top curve); $r_1 = 0.1$ in all eight cases.

(b) $r_1 = 0.1$ (the bottom curve); 0.2; 0.3; 0.4; 0.5; 0.6; 0.7; 0.8 (the top curve); $a_1 = -1$ in all eight cases.

Figure 1: The square brackets factor in the Parisian ruin probability (30) as a function of the claim size rate $\alpha$ for the model from Example 1, Case 1 with $c = 1$, $\lambda = 0.1$, $n = 2$, $r_2 = 10$.

# The End

(for today)