Computable thin Boolean algebras

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Let T be a theory. Its *principal extensions* are extensions of the form $T[\varphi] = \mathrm{Th}(T \cup \{\varphi\}) = \{\psi \text{ is a sentence} \mid T, \varphi \vdash \psi\}$ for some sentence φ.

Let T be a c.e. theory. We say that T *has a few c.e. extensions* if every c.e. extension $T' \supseteq T$ has the form $T[\varphi]$. Note that $T[\varphi]$ is always a c.e. extension of T. This notions was investigated in [1], where some results were proved for essentially undecidable theories with that property, and in other articles.

Below, we consider decidable theories T only. Let $B(T)$ be the Lindenbaum-Tarski algebra of the theory T (it is a computable Boolean algebra).

<u>Note</u>. (1) extensions of the theory T correspond to ideals of $B(T)$

(2) c.e. extensions of T correspond to c.e. ideals of $B(T)$

(3) T has a few c.e. extensions $\Leftrightarrow$ every c.e. ideal in $B(T)$ is principal (one-generated).

We say that a computable Boolean algebra B is *thin* if every c.e. ideal in B is principal (this notion was suggested in [2]). The following theorem describes the algebraic structures of thin computable Boolean algebras.

<u>Theorem</u>. Let B be a computable Boolean algebra. Then the following are equivalent:

(1) $B \cong B'$, where B' is a thin computable Boolean algebra;

(2) B is an atomic Boolean algebra.

References

[1] Downey R.G., *Maximal theories*, Annals of Pure and Applied Logic 33, N3, 1987, 245–282.

[2] *Handbook of Recursive Mathematics*, Elsevier, 1998.