Safety Properties Verification for Pfaffian Dynamics

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We investigate the behavior of a Pfaffian dynamical system with respect to invariants which formalise safety properties. We study continuous dynamical systems which are called Pfaffian, and first introduced in [5, 6]. These systems are defined by Pfaffian functions, either implicitly (via triangular systems of ordinary differential equations) or explicitly (by means of equations and inequalities involving *Pfaffian functions*). Such functions naturally arise in applications as real analytic solutions of triangular first order partial differential equations with polynomial coefficients, and include polynomials, algebraic functions, exponentials, and trigonometric functions in appropriate domains. Pfaffian functions form the largest natural class of real analytic functions which have a uniform description and an explicit characterisation of complexity of their representations in terms of *formats*.

One of the important problems in the theory of dynamical systems is understanding of the behavior of a dynamical system with respect to safety properties. In other words it would be desirable for a given dynamical system to verify a safety property which states that "something bad does never happen", for examples, the power plant will never blow up, the reactor temperature will never exceed $100^o\ C$.

In mathematical settings this problem is formalised in the following way. We consider a continuous dynamical system $\gamma : G_1 \times T \to G_2$, where $G_1 \subseteq \mathbb{R}^{k_1}$ is a set of control parameters, T is an interval of time and $G_2 \subseteq \mathbb{R}^{k_2}$ is a state space. Let U be a set of control parameters. A safety property is formalised by an invariant. An invariant is given by a condition Φ for the states and requires that Φ holds for all reachable states under the control U, i.e. $\forall \mathbf{x} \in U \forall t \in T \Phi(\gamma_{\mathbf{x}}(t))$. In this case we say *the subset* $U \subseteq G_1$ *satisfies the invariant* Φ. We assume that dynamical systems and sets we are interested in are semi-Pfaffian. Our goal is to characterise the subsets of control parameter space which satisfy a given invariant. In order to achieve our goal we use encoding trajectories of a Pfaffian dynamical system by finite words [2, 5] and cylindrical cell decomposition for semi-Pfaffian sets [7, 3]. Based on this technique we construct an algorithm for safety properties verification for Pfaffian dynamical systems with an elementary exponential upper bound.

References

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