SYMMETRIC AND COMPLETE ARCS: BOUNDS AND CONSTRUCTIONS

Fernanda Pambianco

Università degli Studi di Perugia, Italy

collaborators: A.A. Davydov, G. Faina, S. Marcugini, A. Milani

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SUMMARY

- GENERAL CONSTRUCTION OF
  A FAMILY OF ARCS IN $PG(2,q)$
  HAVING $\frac{q+3}{2}$ POINTS ON A CONIC

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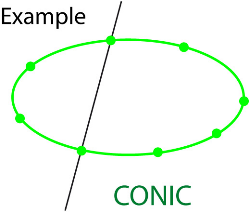
- GEOMETRIC DESCRIPTION OF SOME *EXCEPTIONAL ARCS* AND THE FERMAT CURVE
- NEW SIZES ON THE SPECTRUM OF COMPLETE ARCS IN $PG(2,q)$. THE MINIMUM ORDER OF COMPLETE ARCS IN $PG(2,31)$ AND $PG(2,32)$.

Preliminaries

$$PG(2,q)$$

ARC : set $\mathcal{K}$ no 3-collinear points

Example

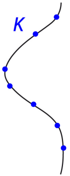


COMPLETE ARC $\mathcal{K}$: $\quad \mathcal{K} \not\subseteq \widetilde{\mathcal{K}}$ arc, $|\mathcal{K}| < |\widetilde{\mathcal{K}}|$

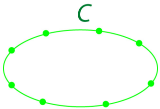
Preliminaries

$\mathcal{K}$: arc in $PG(2,q)$ $\qquad q \geq 5$ odd

$\mathcal{C}$: irreducible conic



$\not\subseteq$



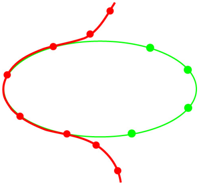
$$\Longrightarrow |\mathcal{K} \cap \mathcal{C}| \leq \frac{q+3}{2}$$

(B. Segre, Ann. Mat. Pure Appl. 1955)

Definition

$$\mathcal{K}_q(\delta):$$

complete $\left(\frac{q+3}{2}+\delta\right)$-arc s.t. $|\mathcal{K}_q(\delta)\cap\mathcal{C}|=\frac{q+3}{2}$



$$\Delta q = max\{\delta\}$$

Open problems

$$\Delta q = ?$$

Examples $\mathcal{K}_q(\delta)$, $\delta \geq 3$?

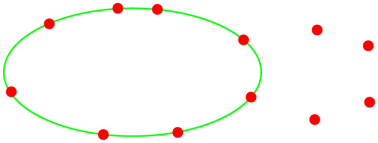
Exceptional arcs

$$(\sim |\mathcal{K}_q(\delta)| \geq \frac{q+9}{2})$$

Constructions of $\frac{q+7}{2}$-arcs, $\Delta q \geq 2$

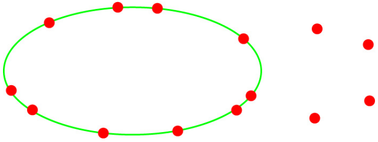
$PG(2,13)$ $\boxed{|\mathcal{K}_{13}(4)| = 12}$ unique

A. H. Ali, J. W. P. Hirschfeld, H. Kaneta, J.C.D. 1994



$PG(2,17)$ $\boxed{|\mathcal{K}_{17}(4)| = 14}$ unique

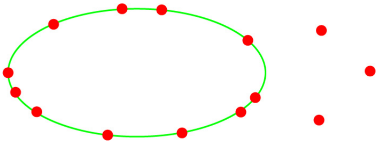
A. A. Davydov, G. Faina, S. Marcugini, F. P., ACCT 2008



$PG(2,19)$ $|\mathcal{K}_{19}(3)| = 14$

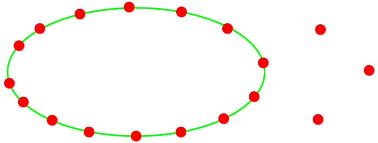
G. Pellegrino Rend. Mat. Appl. (1992)

unique : A. A. Davydov, G. Faina, S. Marcugini, F. P., ACCT 2008



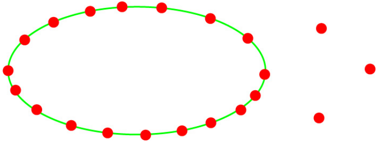
$PG(2,27)$ $|\mathcal{K}_{27}(3)| = 18$ unique

A. A. Davydov, G. Faina, S. Marcugini, F. P., ACCT 2008



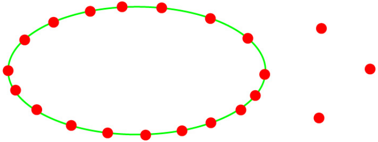
$PG(2,43)$ $|\mathcal{K}_{43}(3)| = 28$

G. Pellegrino Rend. Mat. Appl. (1992)
unique : G. Korchmaros, A. Sonnino, J.C.D. 2009



$PG(2,59)$ $|\mathcal{K}_{59}(4)| = 34$ unique

G. Korchmaros, A. Sonnino, J.C.D. 2009



Theorem (G. Korchmaros, A. Sonnino, J.C.D. 2009)

$$\Delta q = 2 \qquad \begin{cases} q \leq 89 \\ \text{and} \\ q \neq 17, 19, 27, 43, 59 \end{cases}$$

Sporadic examples, $\Delta q \geq 3$ in $PG(2,q)$, $q \equiv 3 \bmod 4$ $(\exists \quad q = 19, \quad q = 43)$ $\dashrightarrow$ Infinite families of examples, $\Delta q \geq 3$ in $PG(2,q^r)$, r odd

General results on Δq and existence of complete $\frac{q+7}{2}$-arcs

- $\frac{q+1}{2}$ prime $\quad \Delta q \leq 4$

  (G. Korchmaros, A. Sonnino, Discr. Math. 2003)

- $q^2 \equiv 1 \mod 16 \quad \Delta q = 2 \quad$ if

  $\frac{q+1}{m} < 1 + \frac{-1+\sqrt{1+(q-1)2\sqrt{q}}}{2\sqrt{q}} \qquad m \neq 1,$ smallest divisor of $q+1$

  (G. Korchmaros, A. Sonnino, J.C.D. 2009)

- $\frac{q+1}{4}$ prime $\quad \Delta q = 2 \quad q \neq 19{,}27{,}43{,}163{,}283{,}331$

  (G. Korchmaros, A. Sonnino, J.C.D. 2009)

- Some constructions of $\frac{q+7}{2}$-arcs, $\Delta q \geq 2$:

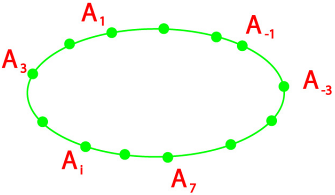
  G. Korchmaros, A. Sonnino, J.C.D. 2009
  V. Giordano, Inn. Incid. Geom. 2009
  A. A. Davydov, G. Faina, S. Marcugini, F. P., J. Geom. 2009
  K. Coolsaet, H. Sticker, Elec. J. of Comb., to appear

Geometric construction

$$q \equiv 2 \mod 3, \quad q = p^n, \quad p \text{ odd prime}, \quad q \geq 11$$

$$\mathcal{C} : \quad x_0^2 + x_1^2 = 2x_0x_2$$

Irreducible conic



$$\mathcal{C} = \left\{ \left(1, i, \frac{i^2+1}{2}\right) | i \in GF(q) \right\} \cup \{(0,0,1)\}$$

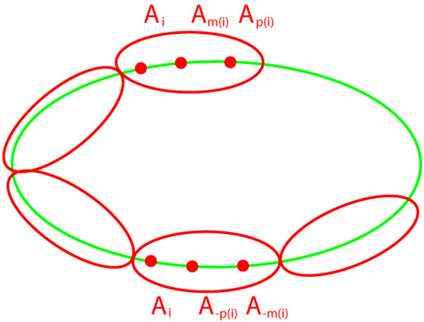
$$A_i = \left(1, i, \frac{i^2+1}{2}\right) \quad i \in \mathbb{F}_q, \quad A_\infty = (0,0,1)$$

Ψ projectivity of order 3

$$\Psi(x_0, x_1, x_2) = (x_0, x_1, x_2) \begin{bmatrix} 0 & -2 & 2 \\ 1 & -1 & -1 \\ 1 & 1 & 1 \end{bmatrix}$$

$\mathcal{C}$ is partitioned into 3-orbits called 3-cycles

$$\mathcal{C}_i = (A_i, A_{m(i)}, A_{p(i)}) \quad \mathcal{C}_{-i} = (A_{-i}, A_{-p(i)}, A_{-m(i)})$$



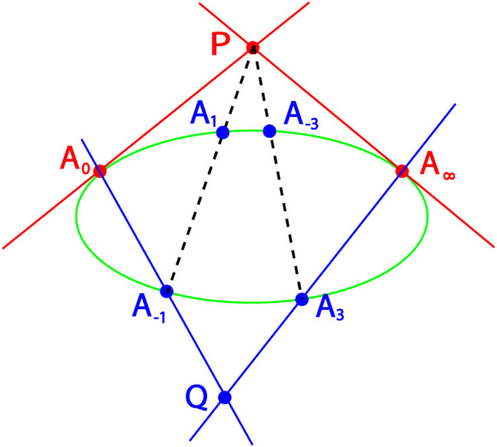
$$m(i) = \frac{i-3}{1+i} \qquad p(i) = \frac{i+3}{1-i}$$

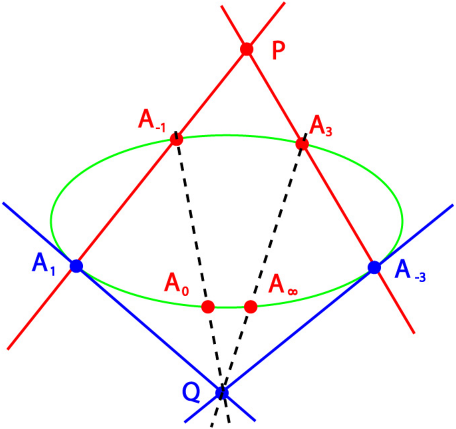
$$\mathcal{C}_i = \mathcal{C}_{-i} \iff i = 0, \pm 1, \pm 3, \infty$$

$$|\{\mathcal{C}_i : \mathcal{C}_i \neq \mathcal{C}_{-i}\}| = \frac{q-5}{3}$$

$P = (0,1,0) \qquad Q = (1,-1,1)$

external points



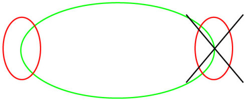


$$\mathcal{B}_6 = \{P, \underbrace{A_0, A_\infty}_{\text{tangent points}}, Q, \underbrace{A_1, A_{-3}}_{\text{tangent points}}\}$$

closed set

$\mathcal{B}_6 = \{P, A_0, A_\infty, Q, A_1, A_{-3}\}$

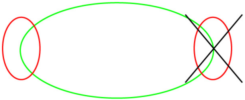
$$\boxed{\mathcal{K}_1 = \bigcup_{i \in \mathcal{I}} \mathcal{C}_i, \quad |\mathcal{I}| = \frac{q-5}{6}} :$$



$\mathcal{C}_i \in \mathcal{K}_1$ $\mathcal{C}_{-i} \notin \mathcal{K}_1$

$\mathcal{B}_6 = \{P, A_0, A_\infty, Q, A_1, A_{-3}\}$

$$\boxed{\mathcal{K}_1 = \bigcup_{i\in\mathcal{I}} \mathcal{C}_i, \quad |\mathcal{I}| = \frac{q-5}{6}} :$$



$\mathcal{C}_i \in \mathcal{K}_1$ $\qquad$ $\mathcal{C}_{-i} \notin \mathcal{K}_1$

Definition

$$\boxed{\mathcal{K} = \mathcal{B}_6 \cup \mathcal{K}_1}$$

$$|\mathcal{K}| = 3\cdot\frac{q-5}{6} + 6 = \frac{q+7}{2} \qquad |\mathcal{K}\cap\mathcal{C}| = \frac{q+3}{2}$$

Theorem

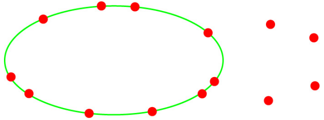
$\mathcal{K}$ is an arc for all the $2^{\frac{q-5}{6}}$ variants of $\mathcal{K}_1$

COMPLETENESS

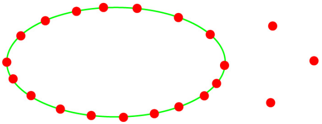
Theorem : $q \equiv 2 \mod 3$, $q = p^n$, p odd prime, $q \geq 11$

- $q \leq 4523$
  $\exists$ one variant of $\mathcal{K}_1$ s.t. $\mathcal{K}$ is complete
- $q \leq 149$ $q \neq 17{,}59$
  all the variants of $\mathcal{K}_1$ make $\mathcal{K}$ complete
- $q = 17{,}59$ $\exists$ non complete examples $\xrightarrow{\text{completing}}$ $\mathcal{K}_{17}(4)$, $\mathcal{K}_{59}(3)$

Geometric description of $\mathcal{K}_{17}(4)$ and $\mathcal{K}_{59}(3)$ as union of *flowers*



$\mathcal{K}_{17}(4)$

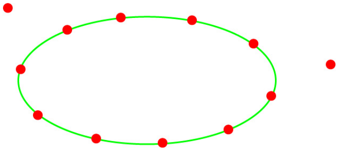


$\mathcal{K}_{59}(3)$

Conjecture: $q \equiv 2 \mod 3$, $q = p^n$, p odd prime, $q \geq 11$

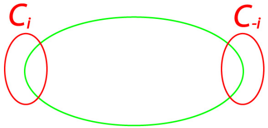
$\exists$ one variant of $\mathcal{K}_1$ s.t.

$\mathcal{K}$ is a $\frac{q+7}{2}$-complete arc



Projective Equivalence

Definition : $\overline{\mathcal{K}_1}$ opposite $\mathcal{K}_1$



$$\mathcal{C}_i \in \mathcal{K}_1 \Rightarrow \mathcal{C}_{-i} \in \overline{\mathcal{K}_1}$$

Proposition

$$\mathcal{K} = \mathcal{B}_6 \cup \mathcal{K}_1 \qquad \widehat{\mathcal{K}} = \mathcal{B}_6 \cup \overline{\mathcal{K}_1}$$

$$\Downarrow$$

$\mathcal{K}$ projectively equivalent to $\widehat{\mathcal{K}}$

$\mathcal{B}_6$ is invariant under

$$G_1 = \begin{bmatrix} 1 & 0 & -4 \\ 0 & 3 & 0 \\ -2 & 0 & -1 \end{bmatrix} \quad G_2 = \begin{bmatrix} 4 & 6 & 2 \\ 3 & -3 & -3 \\ 1 & -3 & 5 \end{bmatrix}$$

$$G_3 = \begin{bmatrix} 0 & -2 & 2 \\ -1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

projectivities of order 2

$$G = \mathbb{Z}_2 \times \mathbb{Z}_2$$

Non equivalent arcs

$$|\mathcal{K}_{neq}| \leq \begin{cases} 2^{\frac{q-17}{6}} + 2^{\frac{q-17}{12}} & q \equiv 1 \mod 4 \\ \lfloor 2^{\frac{q-17}{6}} \rfloor + 2^{\frac{q-11}{12}} - \lfloor 2^{\frac{q-23}{12}} \rfloor & q \equiv 3 \mod 4 \end{cases}$$

For $q = 11, 17, 23, 29, 41, 47, 53$

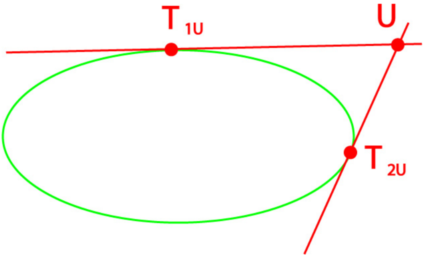
$$|\mathcal{K}_{neq}| = \begin{cases} 2^{\frac{q-17}{6}} - 2^{\frac{q-17}{12}} + 2^h & q \equiv 1 \mod 4 \\ \lfloor 2^{\frac{q-17}{6}} \rfloor - \lfloor 2^{\frac{q-23}{12}} \rfloor + 2^h & q \equiv 3 \mod 4 \end{cases}$$

where $h = \lfloor \frac{q}{12} \rfloor$.

Exceptional Arcs

Lemma:

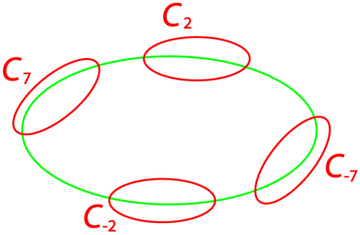
$$\begin{cases} \mathcal{K} = \mathcal{B}_6 \cup \mathcal{K}_1 \\ U \notin \text{ bisecants of } \mathcal{K} \end{cases} \Longrightarrow \begin{array}{l} U \text{ external} \\ T_{1U}, T_{2U} \in \mathcal{K} \end{array}$$

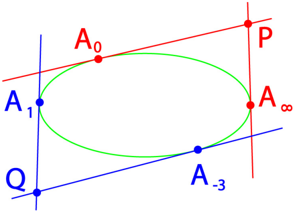


$q = 17$ *exceptional* 14-complete arc

$$\mathcal{K} = \mathcal{K}_1 \cup \mathcal{B}_6$$

$$\mathcal{K}_1 = \mathcal{C}_2 \cup \mathcal{C}_{-7}$$





(equivalent to $\overline{\mathcal{K}} = \underbrace{\overline{\mathcal{K}_1}}_{\mathcal{C}_{-2} \cup \mathcal{C}_7} \cup \mathcal{B}_6$)

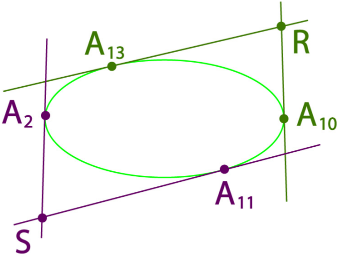
$$|\mathcal{K}| = \frac{q+7}{2} = \frac{17+7}{2} = 12$$

$\mathcal{K}$ incomplete

$$\mathcal{K} \subset \boxed{\mathcal{K} \cup \{\underbrace{R}_{(1,3,6)}, \underbrace{S}_{(1,15,3)}\}}$$

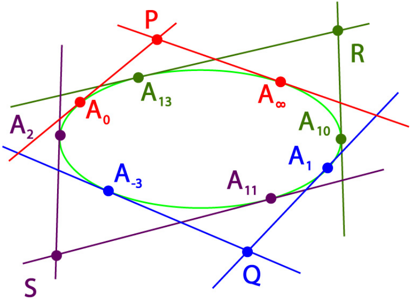
$$\|$$

$$\underbrace{\mathcal{K}_{17}(4)}_{14-\text{complete arc}}$$



$$A_2, A_{11}, A_{10}, A_{13} \in \mathcal{K}$$

Remark



$$\{P, \underbrace{A_0, A_\infty}_{\text{tangent points}}, S, \underbrace{A_2, A_{11}}_{\text{tangent points}}\} \quad \{Q, \underbrace{A_1, A_{-3}}_{\text{tangent points}}, R, \underbrace{A_{13}, A_{10}}_{\text{tangent points}}\}$$

$$\{R, \underbrace{A_{13}, A_{10}}_{\text{tangent points}}, S, \underbrace{A_2, A_{11}}_{\text{tangent points}}\}$$

CLOSED SETS!

(similar to $\{P, \underbrace{A_0, A_\infty}_{\text{tangent points}}, Q, \underbrace{A_1, A_{-3}}_{\text{tangent points}}\}$)

$$\mathcal{C}_i = (A_i, A_j, A_k) \longrightarrow \widetilde{\mathcal{C}}_i = (i, j, k)$$

Respective 3-cycles

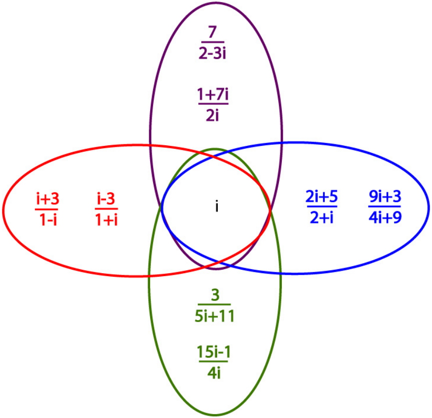
New

$$\begin{cases} \widetilde{C}_i^{PS} = \left(i, \frac{2i+5}{2+i}, \frac{9i+3}{4i+9}\right) \\ \widetilde{C}_i^{RQ} = \left(i, \frac{1+7i}{2i}, \frac{7}{2-3i}\right) \\ \widetilde{C}_i^{RS} = \left(i, \frac{3}{5i+11}, \frac{15i-1}{4i}\right) \end{cases}$$

Previous

$$\widetilde{C}_i^{PQ} = \left(i, \frac{i-3}{i+1}, \frac{i+3}{1-i}\right)$$

Φ_i : Flower with four petals



Proposition:

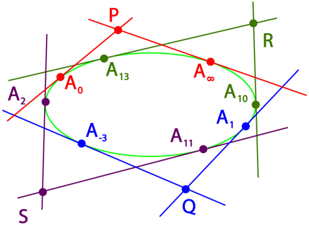
$\forall i \quad \Phi_i$ is a 9-arc.

?

Flowers $\rightsquigarrow$ Structure of $\mathcal{K}_{17}(4)$

Fix external points and their tangent points

$$\mathcal{B}_{12} = \{P, Q, R, S, A_\infty, A_0, A_1, A_{-3}, A_{13}, A_{10}, A_2, A_{11}\}$$



$$\{\underbrace{\mathcal{C}}_{conic} \setminus \mathcal{B}_{12} : \text{non collinear with two points with } \mathcal{B}_{12}\} = \{A_8, A_{12}\}$$

$$\mathcal{K}_{17}(4) = \mathcal{B}_{12} \cup \{A_8, A_{12}\}$$

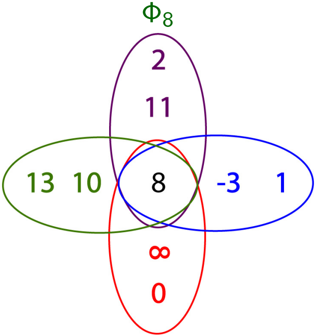
Surprise:

Φ_8 and Φ_{12} have petals formed by

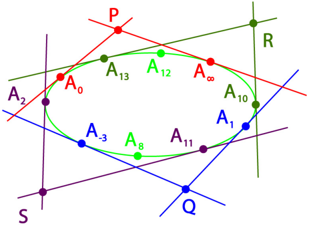
points already chosen

$\sim$

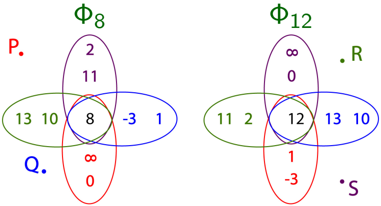
NOT in contrast each other



$$\mathcal{K}_{17}(4) = \mathcal{B}_{12} \cup \{A_8, A_{12}\}$$



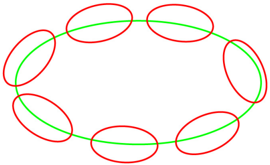
$$\mathcal{K}_{17}(4) = \Phi_8 \cup \Phi_{12} \cup \{P, Q, R, S\}$$

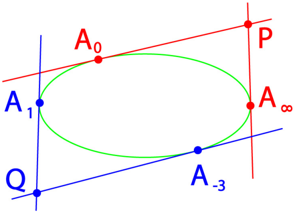


$q = 59$ *exceptional* 34-complete arc

$$\mathcal{K} = \mathcal{K}_1 \cup \mathcal{B}_6$$

$$\mathcal{K}_1 = \mathcal{C}_7 \cup \mathcal{C}_{11} \cup \mathcal{C}_{23} \cup \mathcal{C}_{33} \cup$$
$$\mathcal{C}_{43} \cup \mathcal{C}_{45} \cup \mathcal{C}_{52} \cup \mathcal{C}_{54} \cup \mathcal{C}_{55}$$





$$|\mathcal{K}| = \frac{q+7}{2} = \frac{59+7}{2} = 33$$

$\mathcal{K}$-incomplete

$$\mathcal{K} \subset \boxed{\mathcal{K} \quad \cup \quad \{\underbrace{R}_{(1,8,7)}\}}$$

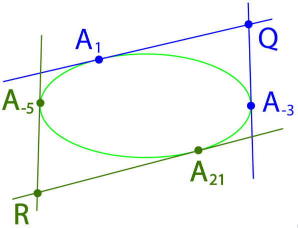
$$\|$$

$$\underbrace{\mathcal{K}_{59}(3)}_{34-\text{complete arc}}$$

Remark

$$\{Q, R, \underbrace{A_1, A_{-3}}_{\text{tangent points}}, \underbrace{A_5, A_{21}}_{\text{tangent points}}\}$$

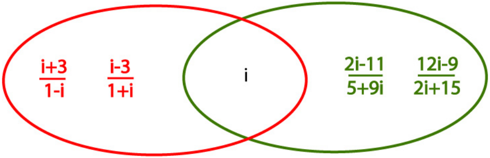
closed set!



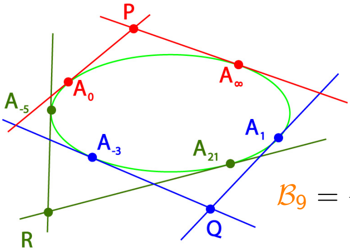
New 3-cycle

$$\widetilde{C}_i^{QR} = \left(i, \frac{2i - 11}{5 + 9i}, \frac{12i - 9}{2i + 15} \right)$$

Φ_i :

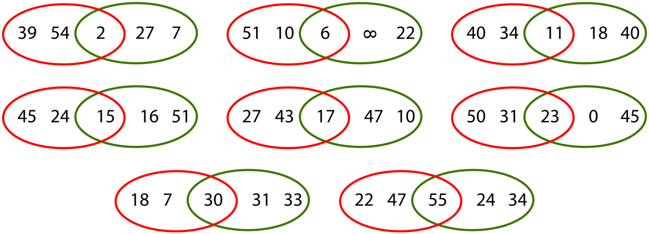


Proposition: $\forall i \quad \mathcal{C}_i^{PQ} \cup \mathcal{C}_i^{QR}$ is a 5-arc.



$$\mathcal{B}_9 = \{P, Q, R, A_\infty, A_0, A_1, A_{-3}, A_{-5}, A_{21}\}$$

$$\mathcal{K}_{59}(3) = \mathcal{B}_9 \quad \cup \Phi_2 \cup \Phi_6 \cup \Phi_{11} \cup \Phi_{15} \cup \Phi_{17} \cup \Phi_{23} \cup \Phi_{30} \cup \Phi_{55}$$



Generalization

$q = 17^3$ 164 flowers 1262-arc $\xrightarrow{\text{completing}}$?

$\frac{q+7}{2} = 2460$

One example of 1874 points

$q = 59^3$ 19566 flowers 80934-arc $\xrightarrow{\text{completing}}$?

$\frac{q+7}{2} = 102693$

What are the planar algebraic non singular curves of degree n
MAXIMALLY SYMMETRIC ?
(proj. autom. group of maximum size)

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$n = 1, 2$		trivial
$n = 3$	$x^3 + y^3 + z^3 = 0 \quad \lvert G \rvert = 54$	F. Enriques and O. Chisini, 1918
$n = 4$	Klein quartic $\lvert G \rvert = 168$ $x^3y + y^3z + z^3x = 0$	R. Hartshorne, Algebraic Geometry 1977
$n = 6$	Wiman sextic $\lvert G \rvert = 360$ $10x^3y^3 + 9(x^5 + y^5)z$ $-45x^2y^2z^2 - 135xyz^4 + 27z^6 = 0$	H. Doi, K. Idei and H. Kaneta, *Osaka J. Math.* 2000
$n \leq 19$ odd prime	$x^n + y^n + z^n = 0 \quad \lvert G \rvert = 6n^2$	H. Kaneta, S. Marcugini and F. P., *Geom. Ded.* 2001

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What happens for general degree n ?

THEOREM

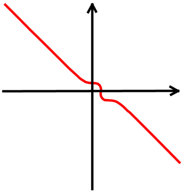
k: algebraic close field

$f \in k[x, y, z]$, homogeneous of deg $n \geq 3$ $n \neq 4, 6$

$V(f)$: non singular algebraic curve in $\mathbb{P}^2(k)$

$$\Downarrow$$

- $|\mathrm{Aut}(V(f))| \leq 6n^2$
- $|\mathrm{Aut}(V(f))| = 6n^2 \iff V(f)$ projectively equivalent to Fermat curve



$$x^n + y^n + z^n = 0$$

Fermat Curve

$x^n + y^n + z^n$

$n|(q+1)$, g genus

$\Downarrow$ Goppa method

code over $GF(q^2)$

of length $N = q^2 + 1 + 2gq$

$\uparrow$

Maximal lenght among algebraic codes

respect Singleton defect $N - k + 1 - d \leq g$

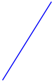
Outline proof of THEOREM

Proposition (H. H. Mitchell, Trans. Amer. Math. Soc., 1911)

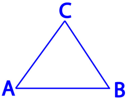
$G \leq PGL(3, k)$

$P \quad \overset{g \in G}{\longmapsto} \quad Q$

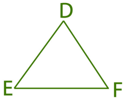
POINT
NO G-invariant

 $\overset{g \in G}{\longmapsto}$ 

LINE
NO G-invariant



$\overset{g \in G}{\longmapsto}$

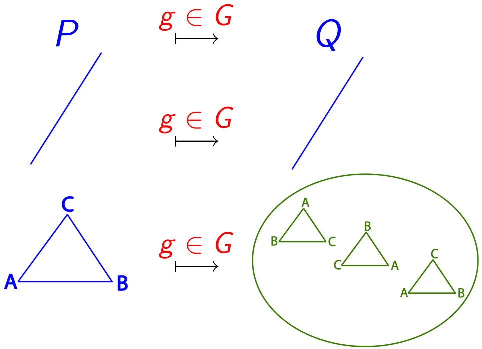


TRIANGLE
NO G-invariant

$$\Longrightarrow |G| \in \{36, 72, 168, 360\} \Longrightarrow \boxed{|G| < 6n^2, \quad n \geq 8}$$

Outline proof of THEOREM

Proposition V(f) : G - invariant, deg $n \geq 11$



G fixes a point

or

G fixes a line

or

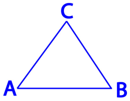
G permutes cyclically

$\Downarrow$

V(f) singular

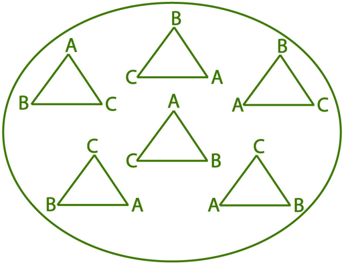
Outline proof of THEOREM

Proposition V(f) : G - invariant, deg $n \geq 11$



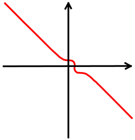
G induces $\mathcal{S}_3$

$\longmapsto$



$\Downarrow$

V(f) non singular $\iff$ V(f) proj. equiv.



Fermat curve

$$x^n + y^n + z^n = 0$$

degree $n = 8$

V(f) G- invariant, $|G| \geq 6n^2$

Hurwitz Theorem $\Longrightarrow 5 \mid |G|$ or $11 \mid |G|$ or $2^7 \mid |G|$

THEOREM

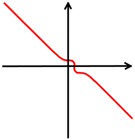
$5 \mid |G|$ or $11 \mid |G|$ or $2^7 \mid |G|$

$\searrow \quad \swarrow \qquad\qquad \swarrow$ or $\searrow$

V(f) singular

V(f) singular

V(f) non singular proj. equiv. Fermat curve



degree $n = 9$

V(f) G- invariant, $|G| \geq 6n^2$

Hurwitz Theorem $\Longrightarrow 2^3 | \ |G|$ or $3^3 | \ |G|$

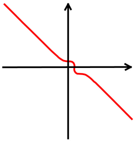
THEOREM

$2^3 \mid |G|$ or $3^3 \mid |G|$

$\downarrow$ — V(f) singular

$\swarrow$ or $\searrow$

V(f) singular — V(f) non singular proj. equiv. Fermat curve



$$\boxed{\textit{degree } n = 10}$$

$$\boxed{V(f) \quad G\text{- invariant,} \quad |G| \geq 6n^2}$$

Hurwitz Theorem $\Longrightarrow 7|\ |G|$ or $13|\ |G|$ or $25|\ |G|$

THEOREM

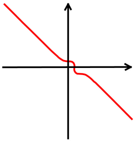
$7 \mid |G|$ or $13 \mid |G|$ or $25 \mid |G|$

$\searrow \quad \swarrow \qquad\qquad \swarrow$ or $\searrow$

V(f) singular

V(f) singular

V(f) non singular proj. equiv. Fermat curve



Small sizes for complete arcs

Definitive Results

MINIMUM ORDER

$$\boxed{t_2(2,31) = t_2(2,32) = 14}$$

Previous definitive result in $PG(2,29)$:
S. Marcugini, A. Milani, F. P., J.C.M.C.C. 2003

Partial Classification of complete 14-arcs

PG(2,31)

G	Non equivalent examples
S_3	1
$\mathbb{Z}_2 \times \mathbb{Z}_2$	4
$\mathbb{Z}_4$	3
$\mathbb{Z}_2$	97
$\mathbb{Z}_1$	3286

PG(2,32)

G	Non equivalent examples
$\mathbb{Z}_5$	1
$\mathbb{Z}_4$	1
$\mathbb{Z}_2$	541
$\mathbb{Z}_1$	8759

Upper bounds for the smallest size $t_2(2,q)$

$$q \leq 841 \qquad \boxed{t_2(2,q) < 4\sqrt{q}}$$

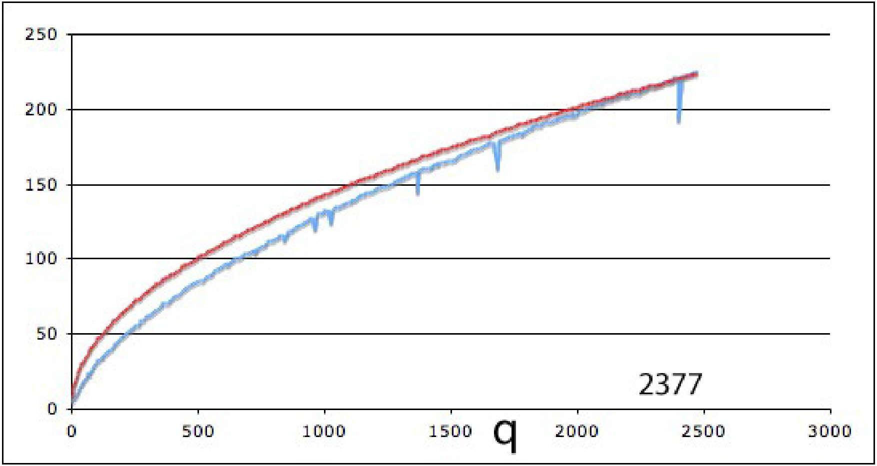
(A. A. Davydov, G. Faina, S. Marcugini, F. P., J.G. 2009)

Results

- $t_2(343) \leq 66$ improvement
- $853 \leq q \leq 6011$ NEW SMALL SIZES

Theorem

$$\begin{aligned}
t_2(2,q) &< 4.2\sqrt{q} & q \leq 1163,\ q = 1181, 1193, 1369, 1681, 2401;\\
t_2(2,q) &< 4.3\sqrt{q} & q \leq 1451,\ q = 1459, 1471, 1481, 1493, 1499,\\
& & 1511, 1681, 2401;\\
t_2(2,q) &< 4.4\sqrt{q} & q \leq 1849,\ q = 1867, 1889, 1901, 1907, 1913,\\
& & 1949, 1993, 2401;\\
t_2(2,q) &< 4.5\sqrt{q} & q \leq 2377,\ q = 2401, 2417, 2437.
\end{aligned}$$



$4.5\sqrt{q}$
SMALL SIZES $= \bar{t}_2(2, q)$

New results on spectrum of complete arcs

$169 \leq q \leq 5171$: NEW SIZES

($q \leq 167$: A. A. Davydov, G. Faina, S. Marcugini, F. P., J. Geom. 1998, 2005 and 2009)

Theorem

In $PG(2,q)$ $\quad 25 \leq q \leq 349$, $q \neq 256$, $q = 1013, 2003$

$\exists$ complete arcs of all sizes in $[\, \bar{t}_2(2,q), M_q]$

where $\bar{t}_2(2,q)$ in previous graphic and

$$M_q = \begin{cases} \frac{q+4}{2} & q \text{ even} \\ \frac{q+7}{2} & \begin{cases} q \equiv 2 \mod 3 & q \text{ odd} \quad 11 \leq q \leq 3701 \\ q \equiv 1 \mod 4 & q \leq 337 \\ \text{other types of } q & \textit{see} \quad \text{G. Korchmaros, A. Sonnino, J.C.D. 2009} \end{cases} \\ \frac{q+5}{2} & \textit{elsewhere} \end{cases}$$

CONJECTURE

In $PG(2, q)$, q prime odd, $353 \leq q \leq 2879$

$\exists$ complete arcs of all sizes in $[\ \bar{t}_2(2, q), M_q]$

Thank

for your attention!