About metrical rigidity of some classes of codes

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Definitions

F_q^n – n - dimensional metric space over the Galois field *GF(q)*, $q = p^r$, with the Hamming metric.

$(n, M, d)_q$ – *parameters* of a q-ary code *C*,

n - length of codewords,

M - size of the code,

d - code distance.

Definitions

An isometry I from *C* to *D = I(C),* $C, D \subseteq F_q^n$, – a mapping *I:C→D,* where *d(x,y)=d(I(x),I(y))* $\forall x, y \in C$.

Equivalent codes – codes *C* and *D* from F_q^n: there exists an isometry π of F_q^n : D = π(C).

Equidistant code – a code C with the distance *d*: *d(x,y)* $= d \quad \forall x, y \in C$.

Definitions

X – a finite set of points of size *v;*

D – a finite family (of size *b*) of *k*-element subsets of *X;*

$\Downarrow$

D – a *design* with *parameters (v, b, r, k, λ)* (or *2-(v, k, λ)-design)* if every element of X is contained in precisely *r* blocks of D and every 2-subset of X is contained in precisely *λ blocks of D*.

A parallel class in *2-(v, k, λ)* design with *v≡0 (mod k)* – a set of *v/k* pairwise disjoint blocks.

Definitions

A resolvable design – a *2*-(v, k, λ)-design: the block set can be partitioned into *r* disjoint parallel classes.

An affine resolvable design – a resolvable design: any two blocks from different parallel classes meet in a constant number elements.

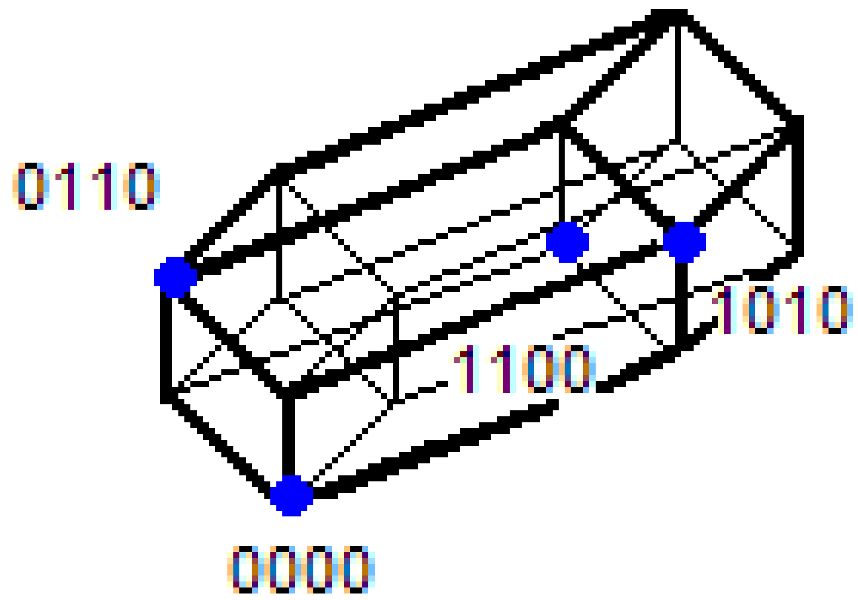
Definitions

A weakly metrically rigid code – a code $C \subset F_q^n$: every code $D = I(C)$ is equivalent to the code C.

A metrially rigid code – a code $C \subset F_q^n$: every isometry $I : C \to F_q^n$ is extendable to an isometry (an automorphism) of F_q^n.

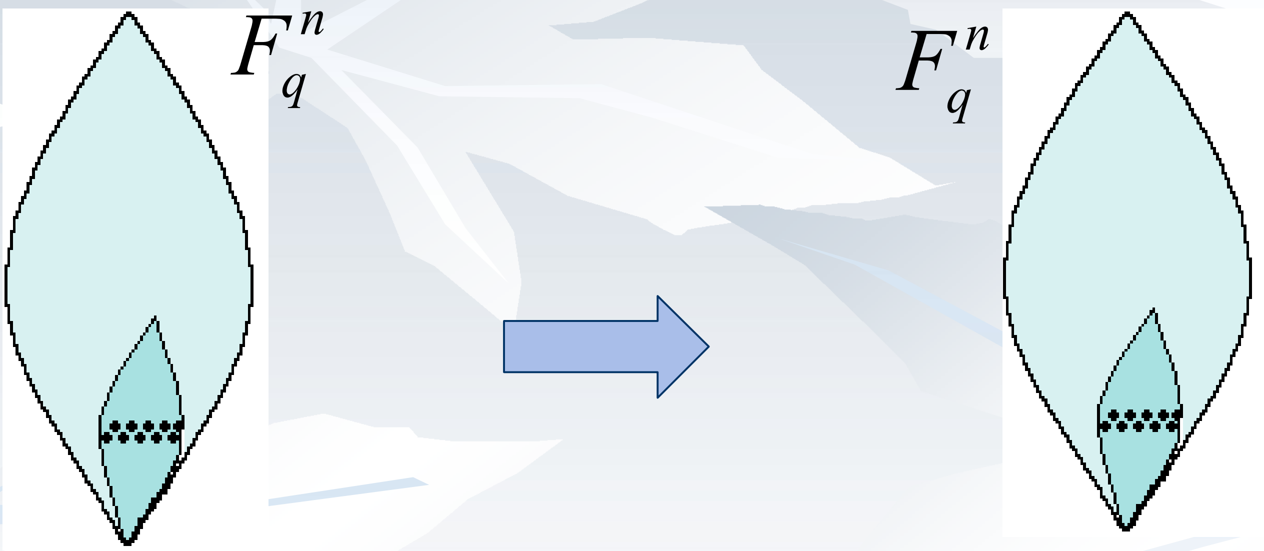
A code, metrically rigid in narrow sense – a code $C \subset F_q^n$: for every $I : C \to C$ there exists an isometry I' of F_q^n : $I\big|_C = I'\big|_C$.

Example



Methods of investigation of metrical rigidity of codes.

1.



2. Finding nonequivalent codes with the given parameters.
3. Comparison of the degree of automorphism group of the code and the number of all isometries, which map this code into itself.

Lemma 1

If the number of all isometries of any q-ary code C, which map this code into itself, is greater then the degree of the automorphism group of this code, then the code C is not metrically rigid in narrow sense.

Lemma 2

An equidistant code C from F_q^n with parameters $((q^2\mu-1)/(q-1), q^2\mu, q\mu)_q$, where $q \geq 3$, $\mu \geq 1$, is not metrically rigid in narrow sense.

Example

```
100 100 100 100
100 010 010 010
100 001 001 001

010 100 001 010
010 010 100 001
010 001 010 100

001 100 010 001
001 010 001 100
001 001 100 010
```

```
0000
0111
0222

1021
1102
1210

2012
2120
2201
```

Lemma 3

An equidistant code C from F_q^n with parameters $(q,(q-1)^2,q-1)_q$, where $q \geq 5$, the numbers q and $q-1$ are some powers of prime numbers is not metrically rigid in narrow sense.

Lemma 4

A code, corresponding to affine resolvable design with parameters *(n,qs,s,q**μ**,**λ**)*, where $n = q^2\mu, q \geq 3, s \geq n \cdot c_q^{\mu}$, $c_q^{\mu} = \log_2 n / \log_2 (n \cdot q!)$, is not metrically rigid in narrow sense.

Lemma 5

Let $k \geq 1$, $\sqrt{6k^2 + 4nk} - 4k < d \leq n/2, d \equiv 0 \pmod{2}$. Then there exist at least $k+1$ nonequivalent equidistant codes with parameters $(n, [2n/d], d)_2$.

Example

0 ... 00	...	...	...	...	0 ... 00	0 ... 00	0..0
1 ... 11	1 ... 11	0 ... 00	0 ... 00	...	0 ... 00	0 ... 00	0..0
1 ... 11	0 ... 00	1 ... 11	0 ... 00	...	0 ... 00	0 ... 00	0..0
...	...	...	...	...	...	...	...
1 ... 11	0 ... 00	0 ... 00	0 ... 00	...	1 ... 11	0 ... 00	0..0
1 ... 11	0 ..011	0 ..011	0 ..011	...	0 ..011	1.10..0	0..0
d/2	d/2	d/2	d/2		d/2	d/2	n - [2n/d]d/2

Theorem

Equidistant codes with parameters:

a) $((q^2\mu - 1)/(q-1), q^2\mu, q\mu)_q$, where $q \geq 3, \mu \geq 1$;

b) $(q,(q-1)^2, q-1)_q$, for $q \geq 5$, where q and $q-1$ are prime powers;

c) $(n,[2n/d],2)_2$, where $\sqrt{16+4n} - 4 < d \leq n/2,\ d \equiv 0 (\mathrm{mod} 2)$;

as well as the binary codes, corresponding to affine resolvable designs with parameters *(n, qs, s, q**μ**, λ)*, where $n = q^2\mu, q \geq 3, s \geq n \cdot c_q^\mu, c_q^\mu = \log_2 n / \log_2(q! \cdot n)$, are not metrically rigid.

Proposition 1

Equidistant codes with parameters $(n, q, 3)_q$, where $n \geq 4$, $q \geq 10$ and $(6,6,4)_3$, as well as the first order Reed – Muller code *RM(1,m)* and punctured first order Reed – Muller code *RM*(1,m)*, are not metrically rigid.

Proposition 2

The Nordstrom – Robinson code, the first order Reed – Muller code *RM(1,m)*, the punctured first order Reed – Muller code *RM*(1,m)*, as well as the equidistant codes with parameters $(n, [n/2], 4)_2$, where $n \geq 17$, and $(6,4,4)_3, (6,4,4)_4, (5,6,4)_3, (6,7,4)_3$, are weakly metrically rigid.

Proposition 3

Equidistant codes with parameters $(n,[2n/d],d)_2$, where $\sqrt{16+4n}-4<d\le n/2, d\equiv 0 \pmod 2)$, as well as equidistant codes with parameters $(n,q,3)_q$, where $n \geq 4$, $q \geq 10$ and $(6,6,4)_3$ are not weakly metrically rigid.

Thank You for the attention!