Majority and Erasure Insertion Decoding of Binary LDPC codes

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LDPC code structure

$$\mathbf{H}_b = \left(\underbrace{\begin{matrix} \mathbf{H}_0 & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{H}_0 & \dots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \dots & \mathbf{H}_0 \end{matrix}}_{b} \right)$$

$$\mathbf{H} = \begin{pmatrix} \mathbf{H}_1 \\ \mathbf{H}_2 \\ \vdots \\ \mathbf{H}_\ell \end{pmatrix} = \begin{pmatrix} \pi_1(\mathbf{H}_b) \\ \pi_2(\mathbf{H}_b) \\ \vdots \\ \pi_\ell(\mathbf{H}_b) \end{pmatrix}$$

$\pi(\mathbf{H}_b)$ - random permutation of $\mathbf{H}_b$ columns

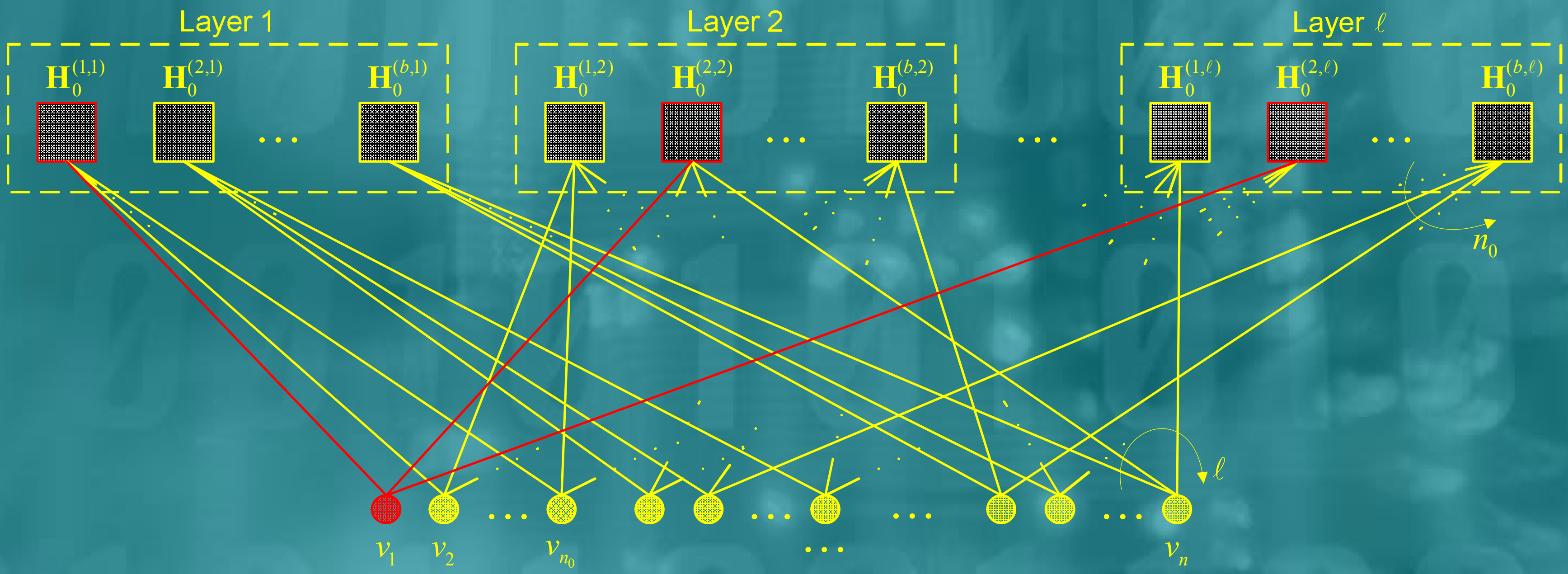
Single parity-check code

$$\mathbf{H}_0 = \underbrace{(111\ldots1)}_{n_0}$$

$$d_0 = 2$$

Corrects any combination of $d_0 - 1 = 1$ erasure

LDPC code with one error



Majority decoding

The main idea of the original majority decoding algorithm is that on each iteration the symbol is inverted if more than $\ell/2$ constituent codes with not fulfilled checks contain the given symbol.

The algorithm stops if the null syndrome is obtained or no one symbol is inverted.

Original ideas

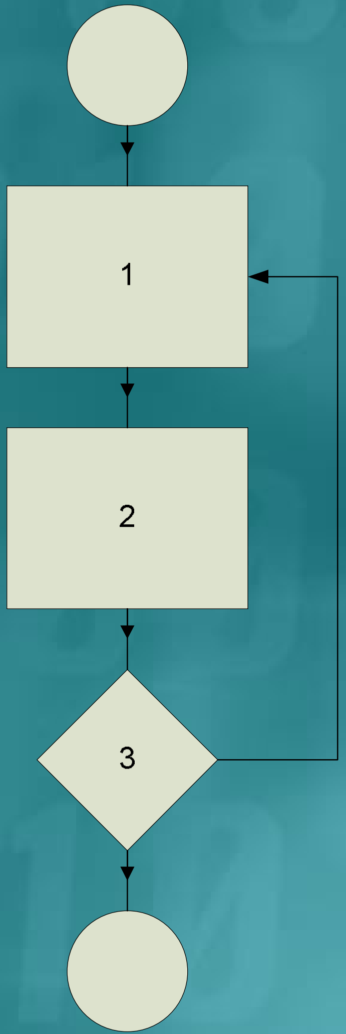
Linear code with code distance d_0:

- corrects any combination of $(d_0 - 1)/2$ errors;
- corrects any combination of $d_0 - 1$ erasures.

So, if code can detect τ errors and correct t errors, then the following is true:

$$\tau \geq t$$

Decoding with erasure insertion



1. Erasure insertion block
2. Erasure correcting block
3. Stop criteria

1. Erasure insertion block

Two erasure insertion criteria:

a) the symbol is erased if more than $\ell/2$ checks are not fulfilled for the given symbol;
b) the symbol is erased if maximum of checks are not fulfilled for the given symbol.

The maximum number of not fulfilled checks is determined by computing the number of not fulfilled checks for each symbol.

2. Erasure correction block

The checks for the given erased symbol are divided into several groups:

a) the checks with not correctable erasure pattern (for example, two erasures for SPC code);
b) the checks with correctable erasure pattern (for example, one erasure for SPC code).

2. Erasure correction block

For each parity-check with one erasure the symbol on erased position is determined by addition modulo two of not erased symbols.

$$v_i = \sum_{j \neq i} v_j \bmod 2$$

Each check with correctable erasure pattern gives decision (1 or 0) for the given erased symbol. We propose to set on the erased symbol the decision that is most frequent.

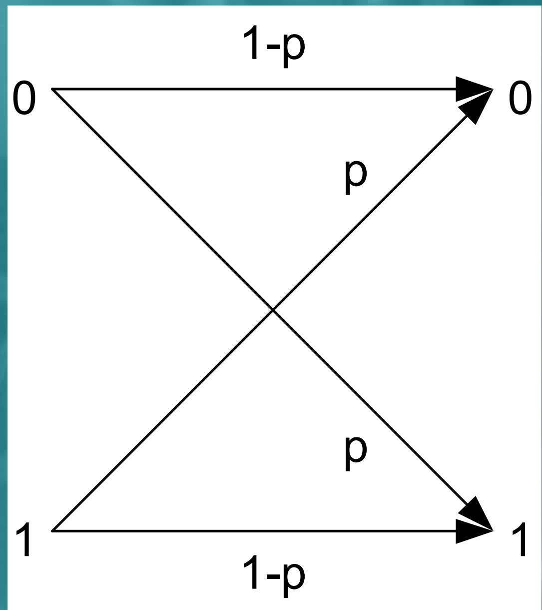
3. Stop criteria

a) Result syndrome is zero and tentative sequence does not contain erasures (success decoding);
b) Result syndrome is not changed with iteration for the second erasure insertion criteria (decoding failure).

The erasure insertion criterion is changed when the previous erasure insertion criterion leads to the decoding failure. The erasure insertion criterion is changed only once during the decoding of one sequence.

Channel with fixed error number

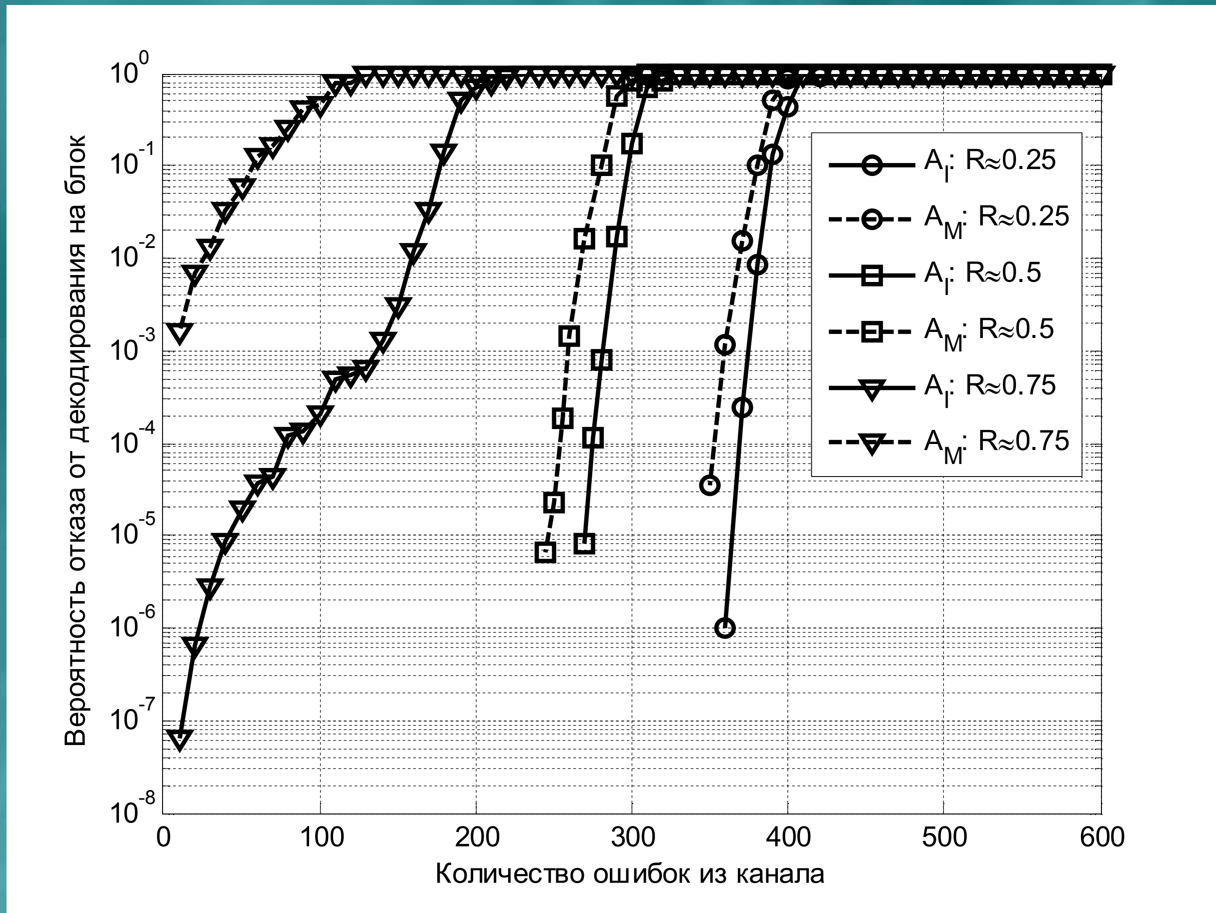
N - number of errors in received code word



$p = N / n$, where n - length of LDPC code

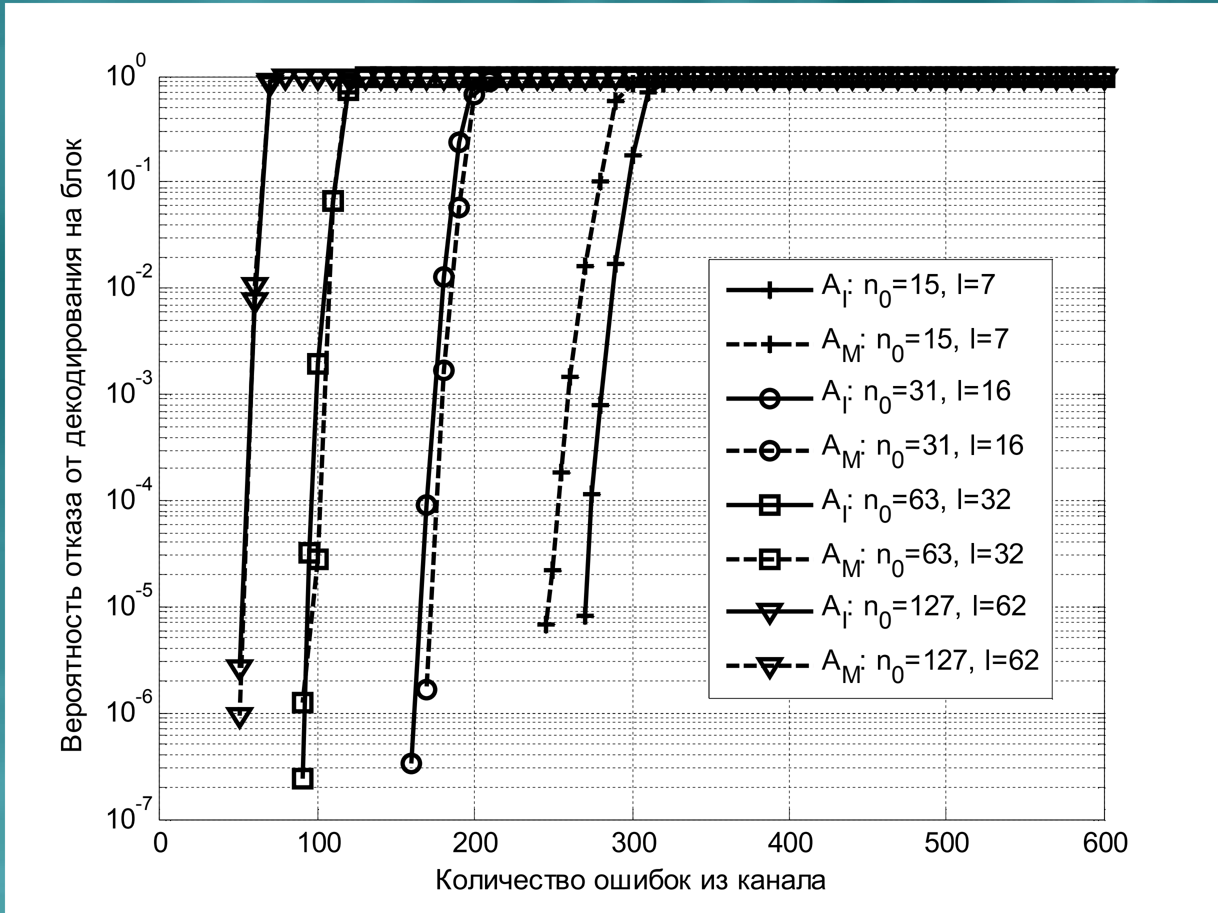
Simulation results

Binary LDPC-codes with $\left(n = 8000, n_0 = 15, R\right)$



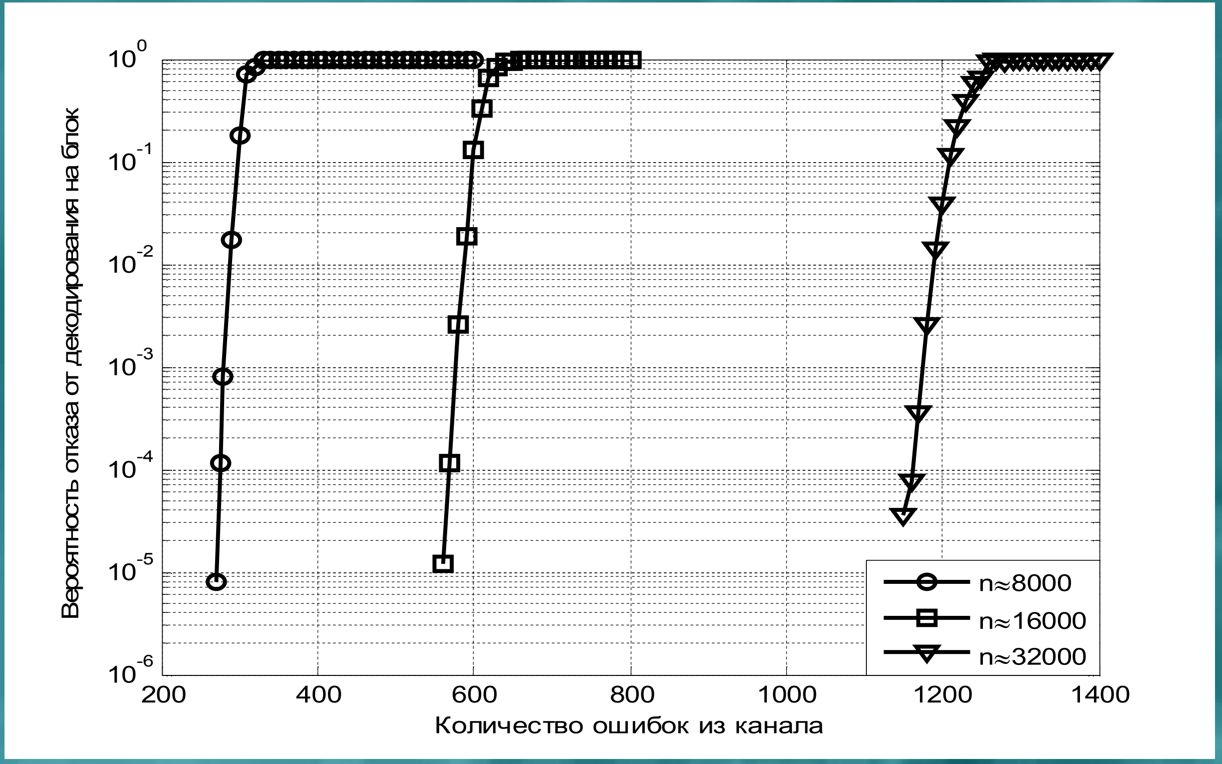
Simulation results

Binary LDPC-codes with $\left(n = 8000, n_0, R \approx 1/2\right)$



Simulation results

Binary LDPC-codes with $\left(n, n_0 = 15, R \approx 1/2\right)$



LDPC code length	8000	16000	32000
Fraction of corrected errors	0.0345	0.0356	0.0363

Thank you!

Any questions?