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Abstracts

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The Dirichlet Problem for SubLaplacians

This talk will be about the Dirichlet problem for the subLaplacian $\mathcal{L} = \sum_{i=1}^{m} X_i^* X_i$ associated to a family of smooth vector fields $X_1, \ldots, X_m$ in $\mathbf{R^n}$ satisfying Hörmander's finite rank condition

$$\text{Rank}\{\text{Lie}[X_1, \ldots, X_m]\} = n.$$

Given a smooth, open and bounded domain $\Omega \subset \mathbf{R^n}$ and a function f defined on $\partial\Omega$, we identify the geometric conditions needed to obtain Poisson type representation formulas for the solution of the boundary value problem:

$$\mathcal{L}u = 0 \text{ in } \Omega$$

$$u = f \text{ in } \partial\Omega$$

We also obtain Lipschitz estimates on the Green function and L^p estimates for the solution u. The geometric conditions depend on the regularity of f. If f is continuous then it is sufficient to require an analogue of the Poincarè outer ball condition, adapted to the Carnot-Caratheodory metric associated to the system $X_1, \ldots, X_m$. If f is in the Lebesgue class we also have to ask that the domain is Non Tangentially Accessible ("NTA" in the sense of Jerison and Kenig) with respect to the Carnot-Caratheodory metric.

One of our main results is the mutual absolute continuity of surface measure and harmonic measure for a large class of domains. This is an analogue of a classical result of Dahlberg. Our proof is based on a reverse Holder inequality for the Kernel function associated to $\partial\Omega$ and on estimates on the harmonic measure in a NTA domain.

We also produce explicit classes of examples of domains satisfying the geometric properties. An important class is given by the balls in the natural gauge metric of a Heisenberg type group.

References

[1] B. E. J. Dahlberg. Estimates of harmonic measure. *Arch. Rat. Mec. An.*, **65**:272–288, 1977.

[2] D. Jerison and C. Kenig. Boundary behavior of harmonic functions in non-tangentially accessible domains. *Adv. Math.*, **46**:80–147, 1982.